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Development of a Technique for Determination of Component Shock Specifications

FINAL REPORT - VOLUME I

Methods for Specifying and Extrapolating Shock Conditions

Report No. 607-4-I

by

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FOREWORD

This report is Volume I of a three volume Final Report presenting the results of work performed by MITRON Research & Development Corporation for NASA Marshall Space Flight Center under Contract No. NAS-8-11090 entitled "Development of a Technique for Determination of Component Shock Specifications". The three volumes comprising the Final Report are as follows:

Volume I	Methods for Specifying and Extrapolating Shock Conditions.
Volume II	Compilation of Four Coordinate Shock and Fourier Spectra for Simple and Complex Shock Motions.
Volume III	Digital Computer Program for Shock and Fourier Spectra.

This project was conducted by the Shock and Vibration Division of MITRON with Mr. Maurice Gertel as Principal Investigator and Mr. Richard Holland as Project Engineer. The program was under the overall cognizance of Messrs Ronald E. Jewell and Thomas Coffin of NASA Marshall Space Flight Center, Propulsion and Vehicle Engineering Division, Structures Branch.

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SYMBOLS

a	equivalent static acceleration	in./sec^2
a	Fourier acceleration component	in./sec^2
A	normalized acceleration	
d	relative deflection response	in.
d	Fourier deflection component	in.
c	damping coefficient	lb-sec/in.
c/c_c	fraction of critical damping	
D	normalized deflection (includes g=386.4 in./sec^2)	in./sec^2
f(t)	function of time	
f_n	undamped natural frequency	cycles/sec (cps)
F	force	lb
F	**Fourier operator**	
g	acceleration of gravity	in./sec^2
G	peak acceleration in number of times gravity	
h	increment of time	sec
Im	imaginary part of	
j	$\sqrt{-1}$	
k	linear stiffness	lb/in.
m	mass	lb-sec^2/in.
n	number	
Re	real part of	
t	time	sec
T	natural period	sec
u	motion of the support	in.

SYMBOLS (Cont.)

$\ddot{U}(\omega)$	Fourier spectrum of $\ddot{u}(t)$	in./sec
$\ddot{U}_C(\omega)$	Fourier cosine spectrum of $\ddot{u}(t)$	in./sec
$\ddot{U}_S(\omega)$	Fourier sine spectrum of $\ddot{u}(t)$	in./sec
v	pseudo velocity response	in./sec
v	Fourier velocity component	in./sec
V	normalized velocity	
x	linear displacement in direction of X axis	in.
Z	impedance	lb-sec/in.
δ	relative response deflection	in.
δ_r	residual relative response deflection	in.
ζ	fraction of critical damping	
θ	phase angle	degrees
τ	period	sec
ω	forcing frequency-angular	rad/sec
ω_n	undamped natural frequency-angular	rad/sec
ω_d	damped natural freqjuencies-angular	rad/sec

VOLUME I

1. INTRODUCTION

Unlike the development of manned vehicles wherein major elements can be performance tested to ascertain required design improvements under actual service conditions, space flight research and defense vehicles are "one-shot" systems and must be laboratory tested for reliability under simulated service conditions. Environmental shock, vibration and acoustic conditions are major factors affecting missile component reliability, hence the development of techniques for accurately measuring, analyzing and interpreting these environments into specifications for design and laboratory test criteria is a subject for continuing attention. The development of valid environmental design and test criteria of necessity requires a closed-loop information feed-back process as typified by the block diagram of Figure 1.

During the past two decades of missile development work, increasing sophistication in instrumentation techniques has produced quantities of data from flight and static firings which have been applied to close the loop depicted in Figure 1 and refine existing acoustic and vibration test criteria. Much of these data have also contained shock transients which could have been analyzed to refine old long standing arbitrary specifications for missile component shock tests. However, lack of an accepted and standardized technique for analyzing transient shock data has hampered and prevented any major examination to refine the development of missile shock test specifications.

The objective of the study reported herein is to establish a theoretically valid method for analyzing missile shock data leading to the development of component shock test specifications. To achieve this objective, current practices for analyzing and specifying environmental shock conditions have been reviewed, with particular emphasis on application to space vehicles and missiles.

It is indicated herein, that missile shock environments are too complex to be satisfactorily defined by a simple wave form, rather shock and Fourier spectral techniques are recommended. A numerical method for shock and Fourier analysis of any arbitrary wave form is presented together with a concept for extrapolation of shock conditions utilizing mechanical impedance properties of components and structures.

It was hoped at the outset of the work described herein, that taped data from static firings of Saturn rocket stages would be available to demonstrate the recommended shock analysis procedures. Subsequently, it was decided that the sample analyses should preferably be performed on recent Saturn flight test data which included pyrotechnic explosive separation of rocket stages. Digital processing of selected portions of these flight data was unfortunately delayed so that analysis of actual flight data could not be included here. However, spectral data from idealized representations of laboratory measured pyrotechnic stage separation shocks are presented. These spectra serve to indicate that most of the current "low frequency" shock specifications with simple pulses and durations, the order of six milliseconds or longer, tend to be unrepresentative of missile shock conditions.

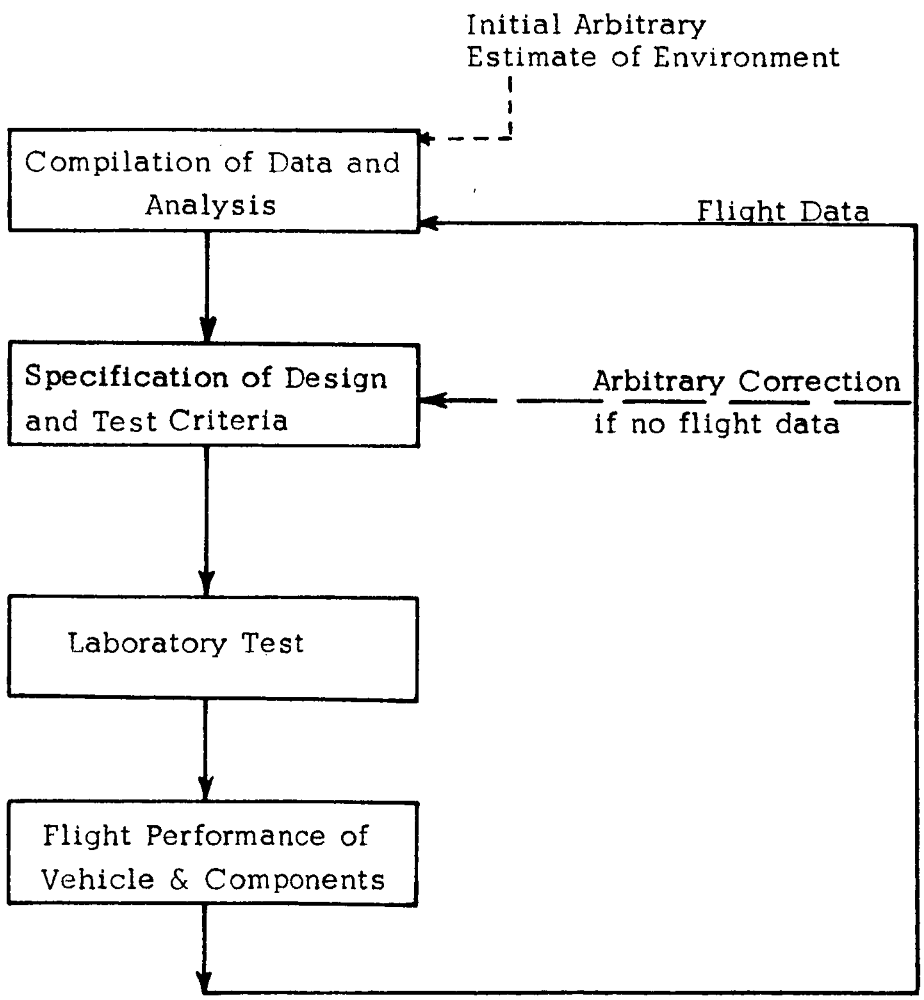


Figure 1 - Closed-Loop Information Feed-Back Process For Development of Design and Test Criteria

2. CONCEPTS FOR SPECIFYING SHOCK ENVIRONMENTS

Historically, there are four methods which have been utilized under varying circumstances for the purpose of specifying environmental shock conditions. These are as follows:

(a) Specific shock machine.

(b) Shock motion time-history.

(c) Fourier spectrum.

(d) Shock spectrum.

The advantages and problems associated with each of the above concepts are reviewed in the following sections.

2.1 Specific Shock Machine

When a shock motion is so complex that it defies description by simple analytic means, then it is sometimes useful to specify a shock testing machine that will produce the same type of damage in equipment as the real environment. This philosophy was of necessity adopted by the Navy during World War II. At that time, combat experience revealed that explosive pressure pulses from underwater mines produced violent shipboard shock motions which did not affect hull integrity, but produced considerable damage to the electronic and electrical equipment essential to the ship's operation. Measurements of these shipboard shock motions indicated magnitudes of acceleration which were unexplainable, particularly since simple stress analysis based upon $\underline{F} = \underline{m}\,\underline{a}$ suggested that the structure which was instrumented should have failed. The urgency of the wartime situation precluded a lengthy analysis of the problem. Hence, an alternative approach was adopted to devise a shock testing machine by trial and error, that would produce similar damage to items under test as that caused during combat shock conditions. The above effort produced the well known Navy High-Impact shock machines for lightweight equipment (up to 400 lbs.) and mediumweight equipment (up to 4500 lbs.) which are still used today to specify combat shipboard shock conditions.

The design and construction of the Navy High-Impact shock machines are completely specified by drawings and each machine in laboratory use is checked for conformance. Furthermore, since the test procedure is simple and quite standardized, this adds to the assurance that equipments will be subjected to uniformly severe qualification shock tests independent of the test technician and the laboratory. The resulting uniformity in shock specification reproduceability has obvious advantages from a contractual point of view, particularly when several organizations may be working on the same or related pieces of equipment.

Notwithstanding the advantage of uniformity which is inherently specified in naval equipment shock tests, designers express considerable dissatisfaction with the apparent need for cut-and-try experience in designing equipment to withstand the Navy High-Impact shock machine tests. In order to counteract this, the Navy has issued numerous technical reports, as for example References 1 to 3, defining in detail the characteristics of the High-Impact shock machines and methods for designing structures and equipment to withstand these tests.

The alternatives of specifying the shock environment rather than a test machine are discussed in the following sections.

2.2 Shock Motion Time-History

The intuitively obvious and direct approach to specifying or describing a shock motion is to define its time-history. The shock motion may be defined in terms of displacement, velocity or acceleration. The particular amplitude parameter which is selected depends on the instrumentation which has been used to measure the shock. In recent years the majority of environmental shock and vibration measurements in missiles have been made with accelerometers because of their small size, light weight and high frequency response.

Defining shock in terms of its time-history is particularly convenient when the shock transient is a simple pulse which can be idealized and expressed by an analytic equation. The majority of current shock specifications in the technical literature have been zealously idealized to permit simple definition. As a consequence, there are numerous commercially available shock test machines which are designed to produce simple half-sine wave, ramp or sawtooth, and rectangular shaped acceleration (or decelaration) pulses, etc.

The principal shortcoming of specifying a motion time-history for shock testing is that the process of "smoothing" or idealizing the shock motion tends to moderate the peak responses of structures with high natural frequencies. This occurs because the process of idealizing inherently eliminates the high frequency components in the shock. Another disadvantage of specifying the shock motion time-history for testing is that this implies a test machine whose test table has infinite mechanical impedance and its motion is not affected by the item being tested. This problem is discussed in References 4 and 5. A further disadvantage is that the frequency response characteristics of instrumentation used to define the shock motion must be indicated so that the performance of different shock machines which might be used can be compared.

If a shock transient embodies a highly complex waveform as is generally the case with most real environments and particularly in missiles, it becomes difficult if not impossible to define the shock motion by a simple analytic equation. Spectral techniques are then indicated. The preferred spectral techniques are described in the following sections.

2.3 Fourier Spectrum

The responses of systems to elementary sine or cosine functions are readily determinable and well known. Hence in determining system responses to arbitrary input functions, it becomes convenient to express the input as a superposition of sine and cosine functions at various frequency components. This process is known as data analysis in the "frequency domain". If the arbitrary input is periodic, its frequency spectrum is defined by the Fourier series. If the input is non-periodic, the spectrum is defined by the Fourier integral. The equations defining Fourier series and Fourier integral are presented briefly here for convenience. Detailed derivations appear in References 6 and 7, for example.

2.3.1 Fourier Series

Periodic functions encountered in vibration analysis may be represented by a Fourier series, expressed mathematically as:

$$f(t) = f(t+T) = \sum_{n=0}^{\infty} a_n \cos n\omega t + \sum b_n \sin n\omega t \tag{1}$$

$$= \sum_{n=0}^{\infty} C_n \sin(n\omega t + \theta_n) \tag{2}$$

$$= \sum_{n=-\infty}^{\infty} C_n e^{jn\omega t} \tag{3}$$

The Fourier coefficients in Equations (1) to (3) are defined by the following equations:

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t) \cos n\omega t \, dt \tag{4}$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t) \sin n\omega t \, dt \tag{5}$$

$$C_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-jn\omega t} \, dt \tag{6}$$

Negative frequencies ($-\underline{n}\omega$) from the combination of Equations (3) and (6) have no physical significance. However, since $e^{jn\omega t}$ and $e^{-jn\omega t}$ represent conjugate pairs of vectors rotating oppositely at the same speed, their vector sum is the real instantaneous value of the desired Fourier harmonic component. Thus, the preferred exponential form of Equation (3) is:

$$f(t) = f(t+T) = C_0 + 2\sum_{n=1}^{\infty} \mathrm{Re}\left[C_n e^{jn\omega t}\right] \tag{7}$$

An example of Fourier series spectrum computation for a periodic square wave is given in Appendix A. The method for computing response of a simple system to this input using the Fourier spectrum is also given.

2.3.2 Fourier Integral

The Fourier integral bears the same relation to the analysis of non-periodic transient functions as the Fourier series does to the analysis of periodic waves. Referring to Equation (3) which is the simplest exponential form of the Fourier series, the term $\underline{c}_n \underline{e}^{jn\omega t}$ represents the sinusoidal component at frequency $\underline{n}\omega$. Now letting period $\underline{T}$ go to ∞, the frequencies $\underline{n}\omega = \underline{n}\,2\pi/\underline{T}$ get closer and closer together so that in the limit when $\underline{T} = \infty$, the frequency components form a continuous distribution. The Fourier series summation thus becomes the Fourier integral:

$$f(t) = \frac{1}{2\pi}\int_{\omega=-\infty}^{+\infty} F(\omega)\, e^{j\omega t}\, d\omega \tag{8}$$

where

$$F(\omega) = \int_{t=-\infty}^{+\infty} f(t)\, e^{-j\omega t}\, dt \tag{9}$$

As a practical matter, consider that the shock starts at $t = 0$ and ends at $t = \tau$, then Equation (9) defining the Fourier spectrum becomes:

$$F(\omega) = \int_0^{\tau} f(t)\, e^{-j\omega t}\, dt \tag{10}$$

An alternate notation for $F(\omega)$ is $F[f(t)]$ which denotes "Fourier transform of $f(t)$". The Fourier spectrum, or frequency spectrum, $F(\omega)$ is a complex number describing amplitude and phase of a sinusoid at frequency ω. $F(\omega)$ can be written in terms of its real and imaginary parts:

$$F(\omega) = [F_c(\omega)] + j\,[F_s(\omega)] \tag{11}$$

where
$$[F_c(\omega)] = \int_{-\infty}^{\infty} f(t) \cos \omega t \, dt \tag{12}$$

$$[F_s(\omega)] = \int_{-\infty}^{\infty} f(t) \sin \omega t \, dt \tag{13}$$

Alternatively, $F(\omega)$ can be written in terms of its absolute value $|F(\omega)|$ and phase angle θ:

$$F(\omega) = |F(\omega)|\, e^{j\theta(\omega)} \tag{14}$$

where
$$|F(\omega)| = \sqrt{[F_c(\omega)]^2 + [F_s(\omega)]^2} \tag{15}$$

$$\theta(\omega) = \arctan \frac{[F_s(\omega)]}{[F_c(\omega)]} \tag{16}$$

Equations (14) and (15) define the Fourier spectrum values which are usually plotted. An example of Fourier integral spectrum computation for a square wave acceleration shock is given in Appendix A. The Fourier spectrum for this pulse is illustrated in Figure 2.

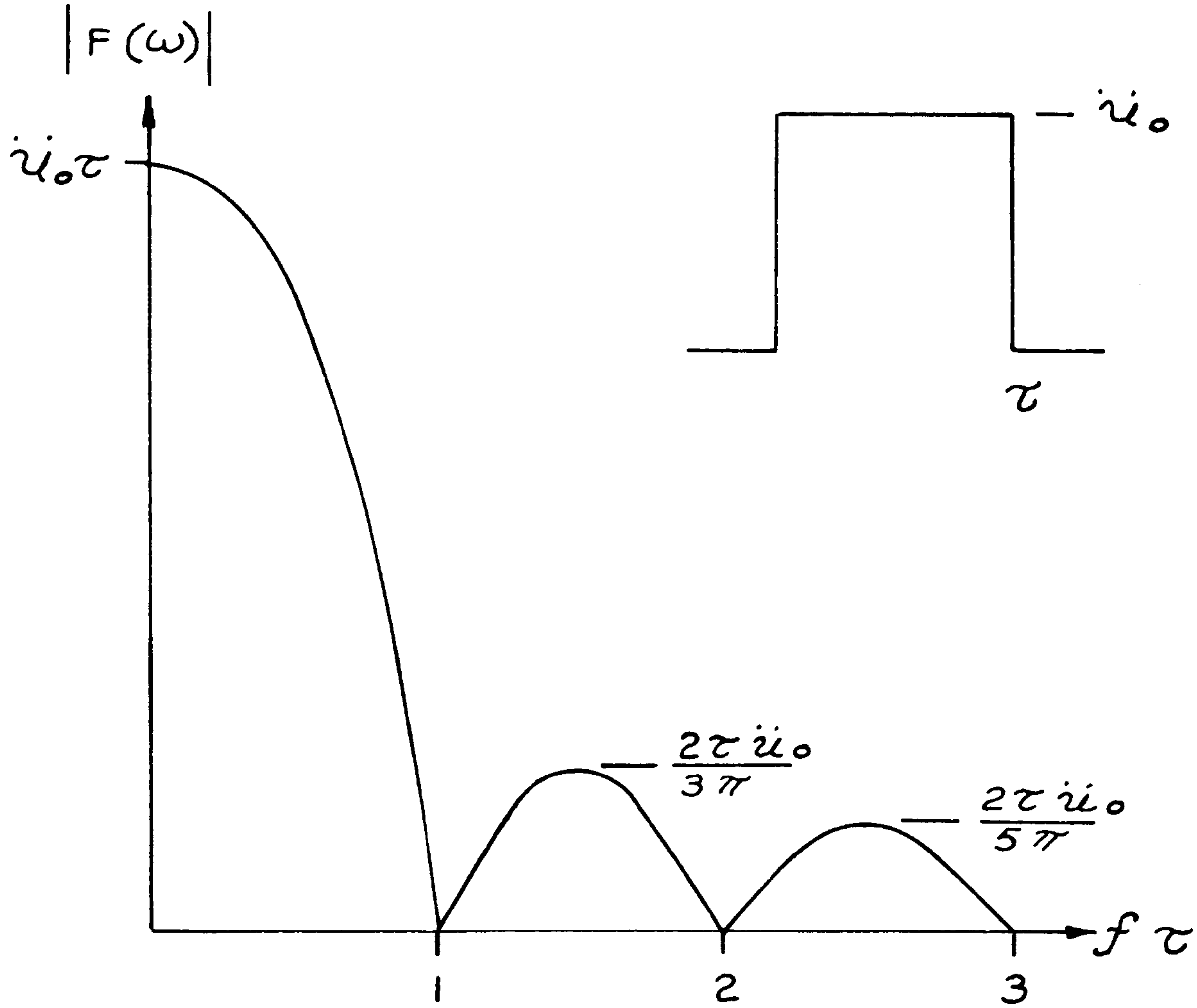


Figure 2 - Fourier Spectrum of Square Wave Acceleration Pulse

2.4 Shock Spectrum

Shock motions can be defined in the "response domain" in addition to the time and frequency domains described in the previous sections. In this concept, each component of an equipment subjected to a shock is conveniently idealized by an undamped single degree-of-freedom system as shown in Figure 3. (The idealized model will be refined to include damping in a later section of this report.)

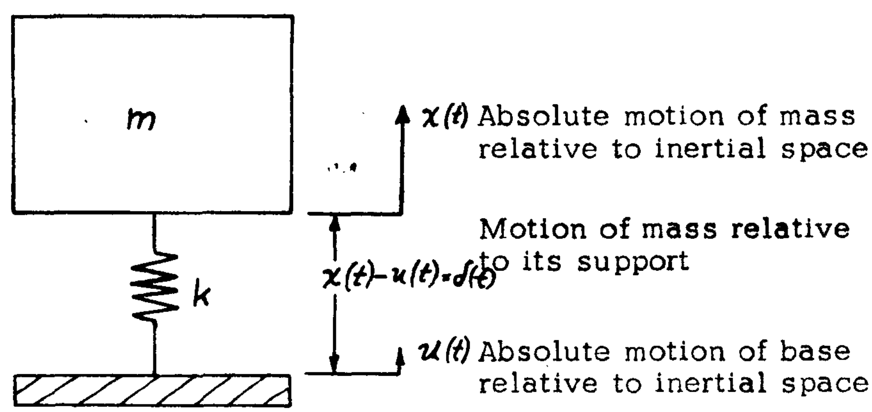


Figure 3 - Idealized Equipment Component

The equation of motion for the system is:

$$m\ddot{x}(t) + k\left[x(t) - u(t)\right] = 0 \tag{17}$$

Rearranging terms, an equation relating absolute response acceleration and relative response deflection is obtained, as follows:

$$\ddot{x}(t) = -\frac{k}{m}\delta(t) = -\omega_n^2\delta(t) \tag{18}$$

The absolute response acceleration of the mass m during a shock input is thus proportional to the relative deflection across the spring portion of the idealized equipment element. Inasmuch as stress in the spring is proportional to deflection, then from Equation (18), the absolute response acceleration of the system is also proportional to the stress. Since damage in a structure is a function of its maximum stress, the damaging potential of a given shock motion may be equivalently stated by the maximum response motion it evokes in simple undamped systems. When the peak response experienced by simple undamped systems with different natural frequencies such as in Figure 3 are plotted as a function of natural frequency, the resulting graph is called the Shock Spectrum of the given input.

The response of an undamped system will be substantially sinusoidal at its natural frequency, hence from Equation (18) the maxima of absolute response acceleration, relative response deflection and velocity can be related as follows:

$$\ddot{x}_{MAX} = \omega_n^2 \delta_{MAX} = \omega_n \dot{\delta}_{MAX} \tag{19}$$

$$\dot{\delta}_{MAX} = \omega_n \delta_{MAX} \tag{20}$$

In the special case of the undamped system considered here, the acceleration, velocity and displacement parameters of Equations (19) and (20) represent true responses. If these are plotted as a function of system natural frequency, the resulting graph could also be termed a "response spectrum".

This terminology is deprecated, however, and the term "shock spectrum" is preferred based upon historical useage. (In the case of the damped system responses to be considered later, the parameters of Equations (19) and (20) will continue to be used. These will not then represent true response acceleration or velocity, and the term "damped shock spectrum" becomes mandatory to signify that a distinction exists.)

In the past decade the shock spectrum has gained increasing acceptance as a method for specifying shock motions. It is particularly useful when it is desired to describe a complex shock motion for simulation in the laboratory. The shock spectrum makes it feasible to consider laboratory simulation of the shock in terms of reproducing the same damaging effects, i.e., peak responses, rather than an accurate reproduction of the shock time-history. Aside from its advantages in connection with specifying laboratory simulation, the shock spectrum provides valuable information for designers because the frequency ranges, wherein resonant systems will experience the greatest peak response, are immediately evident.

There are two general forms of the shock spectrum which appear in the technical literature on shock. These are the two-dimensional shock spectrum and the three-dimensional shock spectrum which are described in detail in References 4 and 8. The appropriate application of these forms of shock spectra depends to a great extent on the severity of the shock encountered. Figures 4 and 5 illustrate two forms of the two-dimensional shock spectrum. Figure 6 depicts the three-dimensional shock spectrum.

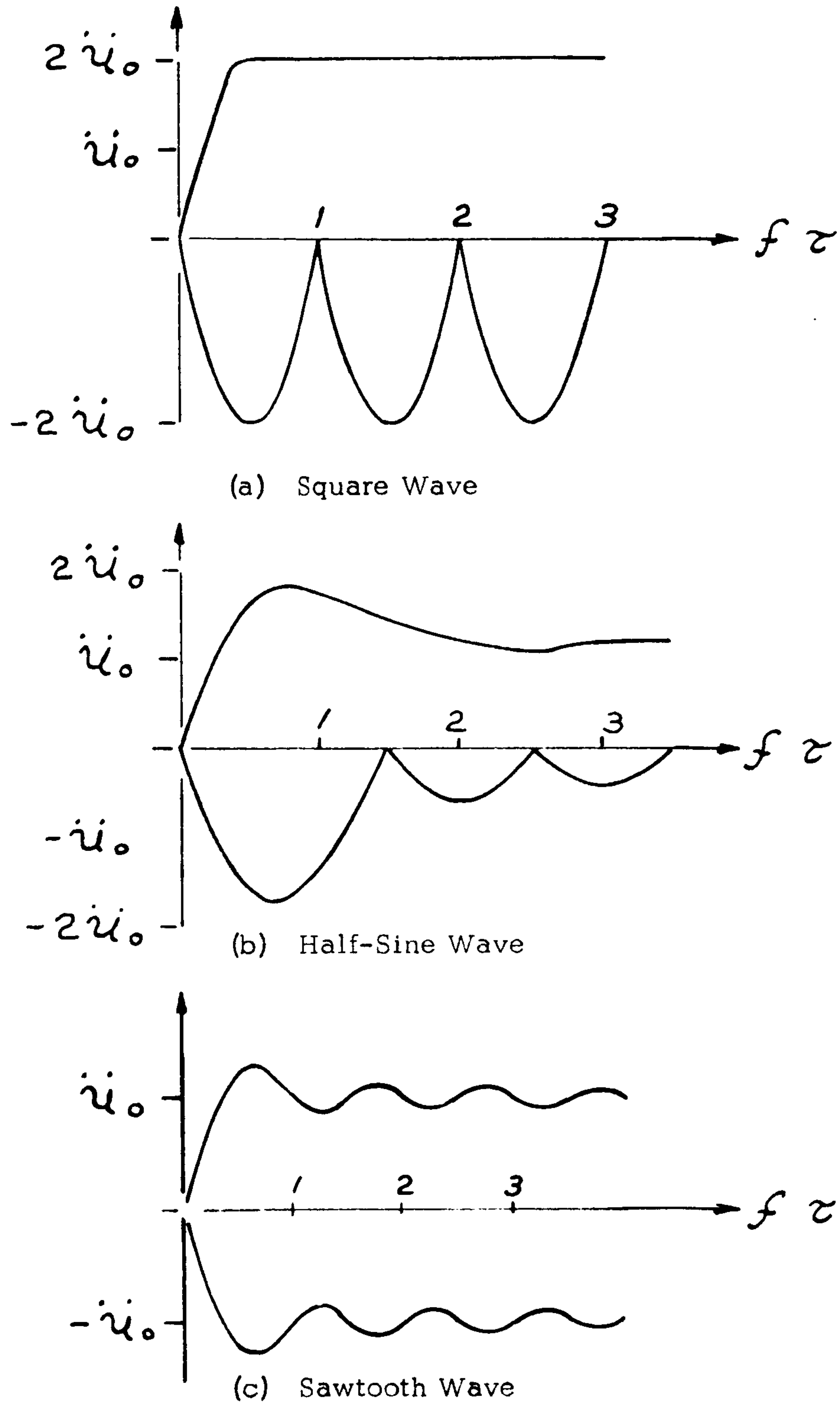


Figure 4 - Two Dimensional Shock Spectra for Acceleration Pulse with Peak Acceleration $\ddot{u}_o$ and Duration τ.

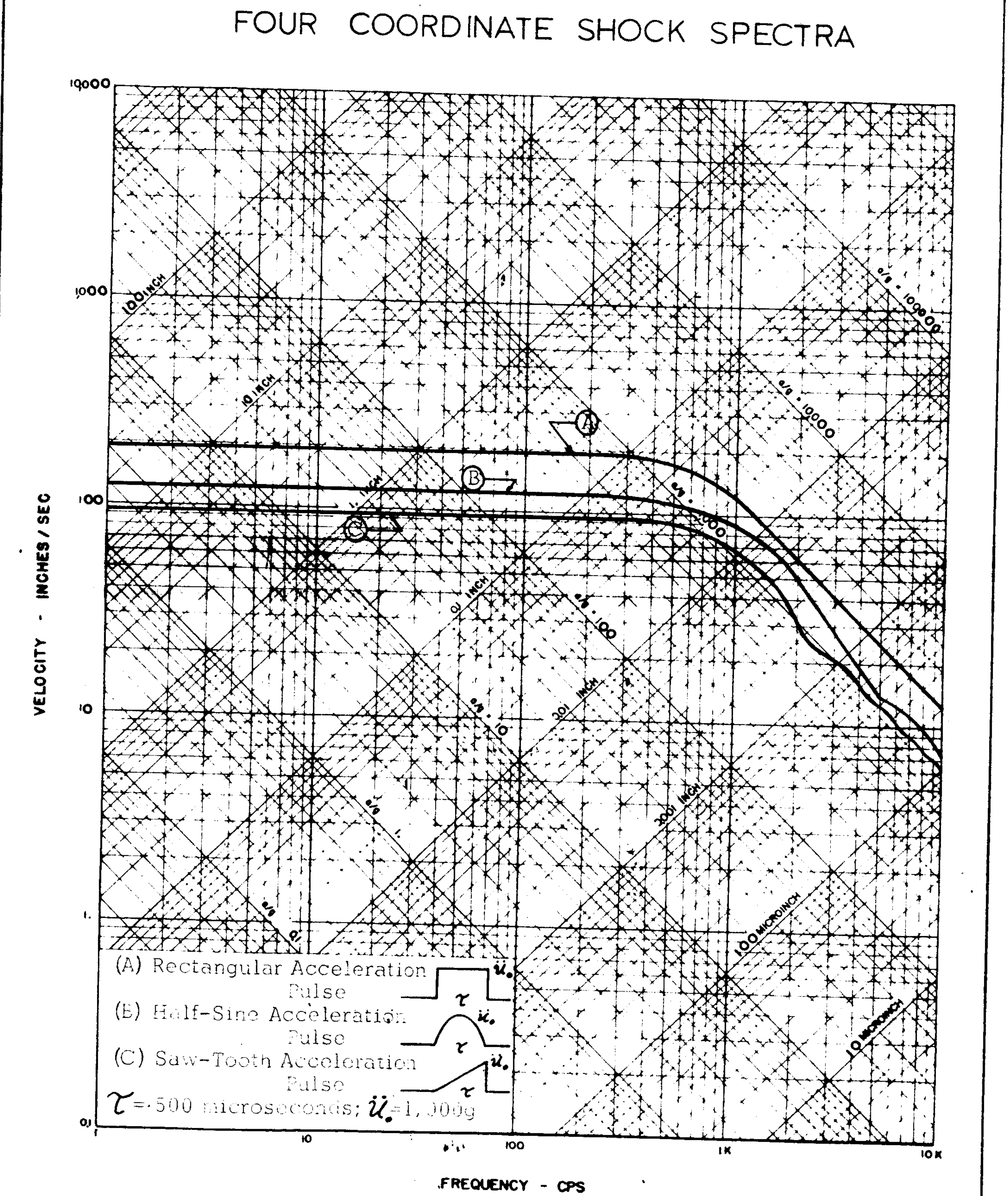


FIGURE 5: Four Coordinate Presentation of Two-Dimensional Shock Spectrum in Figure 4

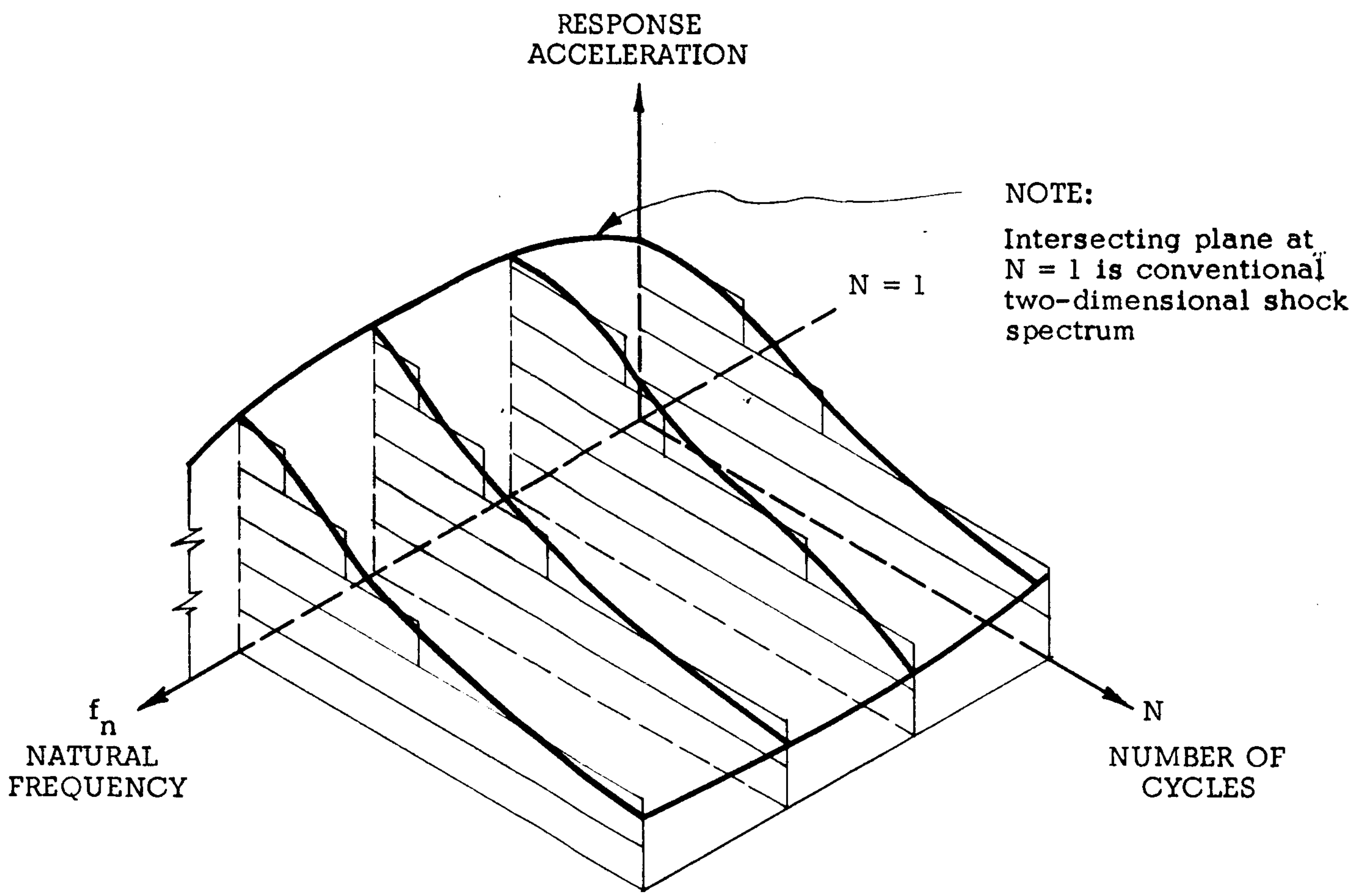


FIGURE 6 - Three-Dimensional Shock Spectrum for Depicting Fatigue Effects

2.4.1 Two-Dimensional Shock Spectrum

The ideal application of the two-dimensional shock spectrum is for describing a shock motion which is so severe that equipment components are required to withstand only one application of the shock. In this case, only the maximum value of the spectrum is of interest. If failure is produced it will be because ultimate stress is developed in the component and not because of a fatigue effect. Since missile shock conditions are not normally of a long term repetitive nature, the two-dimensional shock spectrum is a highly promising technique for application here.

2.4.2 Three-Dimensional Shock Spectrum

The three-dimensional shock spectrum provides a refinement of the two-dimensional spectrum which permits analysis of fatigue effects. This is accomplished by plotting the number of repetitions of various response levels in the form of bar histograms along a third coordinate which is orthogonal to the conventional two-dimensional shock spectrum. The three-dimensional shock spectrum has been used in studies of aircraft landing shocks where fatigue effects from numerous shock repetitions is a major factor. It is doubtful that this approach will be pertinent for missile shock.

3. SURVEY OF PRESENT SHOCK SPECIFICATION PRACTICES

A brief survey of technical literature and personnel in government and industry concerned with the derivation and/or specification of shock tests was conducted during the course of the research reported herein. The brief bibliography on shock in Appendix B summarizes the literature which was reviewed. This survey revealed that the majority of current shock test specifications have been selected by arbitrary means, because specifications and design criteria must be set at an early stage before system flight data are available. In view of this it is practical and less costly to select an arbitrary test condition for which facilities exist, rather than specify arbitrary conditions which will require development of special test equipment. The result is a strong tendency to specify "standard" shock tests with idealized wave forms such as half-sine and terminal peak sawtooth acceleration pulses, etc. The latter shock pulse shape has apparently achieved great acceptance as an arbitrary shock test because it produces essentially equal response in all frequencies, as may be noted from the shock spectrum data in Volume II.

Typical comments received from shock and vibration personnel in government and industry relative to current published shock specifications for missiles are summarized briefly below. These comments are listed by Company names, rather than individuals in order to provide industrial background. The reported remarks are not to be construed as official company policies, but do reflect attitudes which are prevalent.

AF - Aerospace Corp.: Strong advocate of shock spectrum. Currently examining application of Fourier spectra. Usually specifies pulse shape and shock spectrum.

AF - Wright Field: Associated with development of generalized environmental specs and proponent of MIL-Std-810 and MIL-Std-202. These specs define shock pulse shapes.

AVCO: Shock specs are set by customer. Usually these are MIL-specs. Usually shock pulse shape is specified.

Boeing: Shock specs are defined by contracting agency. Usually MIL-spec.

Douglas Missile & Space Systems Div.: Shock specs are defined by contracting agency.

General Dynamics, Electric Boat: Shock specs defined by contracting agency (Navy). Usually shock machine and shock spectrum references are identified.

Grumman Aircraft: Shock specs defined by contracting agency. Usually MIL-E-5400 and MIL-Std-810.

Kearfott: Shock specs for internal components of equipment are arbitrarily set higher than equipment spec. Usually half-sine pulse shocks.

Ling Electronics Div.: Have license agreement with Lockheed for manufacture of shock spectrum analyzer. Interested in application of electromagnetic shakers to reproduce shock spectra.

Lockheed: On Polaris project have done considerable experimental work on application of electromagnetic shakers to produce shock spectra computed from flight shock data. In aircraft projects, contracting agency specifies MIL-spec.

Navy Department: Considerable quantities of shipboard shock measurements have been interpreted into shock spectra for correlation with existing HI-Impact shock machine spectra. Specs to contractors involve definition of measured or scaled shock time-histories, shock spectra and/or shock machine.

Sandia: Prime concern is with nuclear weapon components. Shock specs generally reflect available drop and impulse type machines. General Standards documents define environmental test levels, selection and methods. Field Environmental data is filed for future spec work.

A somewhat similar informal survey of engineering personnel intimately concerned with environmental shock problems was conducted in September 1962, by Wright Aeronautical Systems Division personnel during a coordination conference on MIL-Std-810. A vote was taken on the conference attendees' preferences for a method of specifying shock. Four methods were voted on, with first and second choices indicated as follows:

Type of Specification	First Place Votes	Second Place Votes
Shock Machine	4	3
Shock Spectrum	12	12
Pulse Shape	14	8
Fourier Spectrum	1	3

The above vote does not include the Wright Field conferees , since they were sponsors of the conference.

A critical and objective examination of the results of the above noted surveys, leads to one conclusion. The development of shock specifications in missile systems is exclusively the responsibility of government technical and procurement agencies. In general, industry acts as a supplier of systems and components and will develop these to meet the contractual shock test requirements which are specified. Clearly, the government technical agencies must sponsor appropriate research to define environmental shock environments and then lead in specifying the required shock test criteria in a manner which is technically valid and significant.

It seems to be widely recognized that current specifications for qualification shock testing of missile components are arbitrary, inadequate and improperly specified. The study reported herein was undertaken expressly for the purpose of providing a method of approach for developing shock specifications that will be representative of actual missile environments.

4. CONCEPTS FOR DEVELOPING SHOCK TEST CRITERIA

4.1 Relation Between Shock Test Criteria, Design Criteria and Environment

Throughout this report the terms "shock test criteria" and "shock design criteria" are used synonymously. The reason for this is that in the present practical process of missile component development, the acceptability of components for flight service is contingent upon satisfactory performance during laboratory environmental shock and vibration testing. Ideally, it is desired that these laboratory tests should be representative of service operational conditions, with some margin for safety. However, after the "service environment" has been transformed into the legalistic form of a laboratory test specification which appears as a contract requirement, it makes little or no difference to a potential contractor how accurately the real environment is represented. The contractor's only responsibility is to pass the laboratory test, hence the test automatically defines the design criteria.

Shock test and design criteria must be carefully distinguished from service shock conditions. The definition of particular service shock conditions involves quantities which can be measured very explicitly. In contrast, test and design criteria are a concept which permit an analyst or designer to estimate maximum stresses and strains in a contemplated structure which will result from a set of environmental conditions — not any one given field condition. In order to achieve this conceptual objective, it is essential that test criteria or spectra be related to the measured shock and Fourier spectra or environment time-history in a very special way. Reference 9 describes two currently advocated approaches to defining the relationship between test and measured shock and vibration conditions. These approaches, together with variations conceived during the present program are summarized in the following sections.

4.2 Exact Duplication of Shock Environment

4.2.1 Reproduction of Time-History

Conceptually, it might appear that the optimum in laboratory shock testing would be an exact duplication of the measured environment time-history. If the environment is a simple shock pulse, one of the present commercial shock machines could be adapted. If the shock environment is complex, it is possible that actual tape recordings of flight vehicle shock transients could be obtained for various locations of interest and these could be used as the input to an electronically controlled shock testing machine. This approach is in fact being developed and utilized experimentally as may be noted in References 10 to 12. Widespread application of this approach seems limited, however, for such practical reasons as the following:

1. The environment severity in missile and rocket vehicle flights tends to be statistical in nature. Hence, the available sample records of shock transients may not necessarily be the best representation for testing purposes.
2. The shock environment in a flight vehicle is not expected to be the same at any two locations due to variations in structure and distance from the origin of the shock. Exact duplication of environment would require a different test for each structural location.
3. Practical limitations on flight data acquisition preclude the possibility of obtaining sufficient quantities of shock environment recordings to permit different tests at every component and equipment location in a flight vehicle.

Exact duplication of a shock environment by definition requires that the test severity level and duration should be identical to the real environment. Also, the number of repetitions must be the same. Clearly, these particular factors present no laboratory duplication problems for missile and rocket vehicle applications where service life is extremely short compared to aircraft.

4.2.2 Reproduction of Fourier Spectrum

In the past, when exact duplication of environments has been considered, investigators have approached this from the point of view of exactly reproducing the time-history of the environment. One of the principal problems has been that the accuracy of achieving the desired reproduction is a subjective matter and contingent upon visual detection of possible subtle differences between the real and reproduced environment. It is proposed here that Fourier spectra can be utilized to eliminate the problems normally anticipated in the development of shock tests based on exact duplication. Fourier spectra of the real and reproduced environments can be computed and compared in a very precise manner to obtain a quantitative definition of the accuracy of the reproduced test environment. In this Fourier spectrum approach to exact duplication, the development of the desired test can proceed in a concisely defined manner without reliance on subjective opinions as to the accuracy of the reproduction in the laboratory.

4.3 Simulation of the Damaging Effects of the Environment

Navy experience with shipboard shocks (the complexity of which has much in common with missile shock conditions) in that their most characteristic feature is their infinite variety. Accordingly, for missile shock tests it is possible that it is neither desirable nor practical to construct a shock machine which exactly reproduces a given service condition. Rather, the shock test should have a damage potential as great as any probable service shock to which vital equipment or components will be exposed.

The alternative to exact duplication of a particular shock transient is to synthesize a test based upon an envelope of the most damaging characteristics of several shock transients. Inasmuch as precise duplication possibly involves sophisticated techniques of testing, it is expected that aside from the desirable aspects of having a single test represent a whole set of shock environments, the process of enveloping may create a test condition which can be performed by a simpler testing machine.

4.3.1 Envelope of Shock Spectra

The process of enveloping the most damaging characteristics of shock environments is not new and was developed in detail in Reference 13, using shock spectra to define the damaging potential of shock motions. The criterion adopted therein was that damage in structures and equipment due to shock is attributable to the induced stress response and the number of repetitions. This leads to (1) a maximum stress criterion of damage or failure and (2) a fatigue criterion. In alternative (1), the conventional two-dimensional shock spectrum described in Section 2.4.1 is used to define shock severity. For alternative (2), the three-dimensional form of shock spectrum described in Section 2.4.2 is necessary to provide data on the number of repetitions of peak responses. Due to the short service life of missile and rocket components, fatigue considerations involving design for long life are not pertinent. Hence, the presentation of missile and rocket shock test specifications can be safely restricted to the conventional two-dimensional shock spectrum.

It is an inherent assumption of the enveloping process as applied to shock spectra that the "foundation", i.e., equipment support structure, impedance is very large over the entire frequency range. This implies that installed equipment and components have negligible impedance and will exert no influence on the motion of the support. Obviously this is not true. It has been shown in Reference 14, for example, that there is a general lowering or valley in the shock spectrum values in the region of fixed base natural frequencies of mounted components. This phenomenon has been referred to as "shock spectrum dip". The frequencies at which these spectrum valleys occur are extremely critical with regard to damage because at these frequencies, mounted components tend to act like tuned absorbers and are driven to high vibration amplitudes while the motion of the support is "absorbed" and reduced to negligible amplitude. It is essential that the importance of shock spectrum dip due to foundation-equipment impedance matching be considered when spectral data are enveloped. This is a problem for further detailed analysis. A conceptual approach for resolving this problem utilizing mechanical impedance properties in conjunction with environment Fourier spectra is presented later in this report.

4.3.2 Envelope of Fourier Spectra

By analogy with the concept of enveloping environmental shock spectra, it is reasonable to presume that a composite severe environment for shock testing criteria can also be developed by enveloping environmental Fourier spectra. There is no precedent for this in shock testing technology because the application of Fourier spectra to mechanical shock problems has never been fully exploited. There is, however, a related precedent in that composite severe vibration environments for testing are conventionally defined by envelopes of the well known "fly-speck" plots of Fourier harmonic components.

The concept of enveloping Fourier spectra requires further study. Of particular concern here, is that the enveloping process will tend to eliminate the frequency "notches" which are characteristic in Fourier spectra of symmetrical shaped pulses. This may be seen in Figure 2 and the compilation of spectra presented in Volume II of this report. The effect of Fourier spectrum notches on comparing the relative severity of shock pulses based on their spectra is a related subject which is discussed later.

4.4 Relative Shock Severity Based on Shock Spectra

Inasmuch as shock spectra are plots of the maximum responses evoked from simple systems, it is relatively straight forward, that in comparing two shock motions, the one with the highest spectrum is considered to be the more damaging of the two. This concept is described in Reference 15, for example. If the shock spectra intersect, then each shock motion is respectively considered more damaging than the other in the frequency region where its shock spectrum is the higher of the two. The foregoing points are illustrated in Figure 7 with three rectangular acceleration pulses and their corresponding shock spectra.

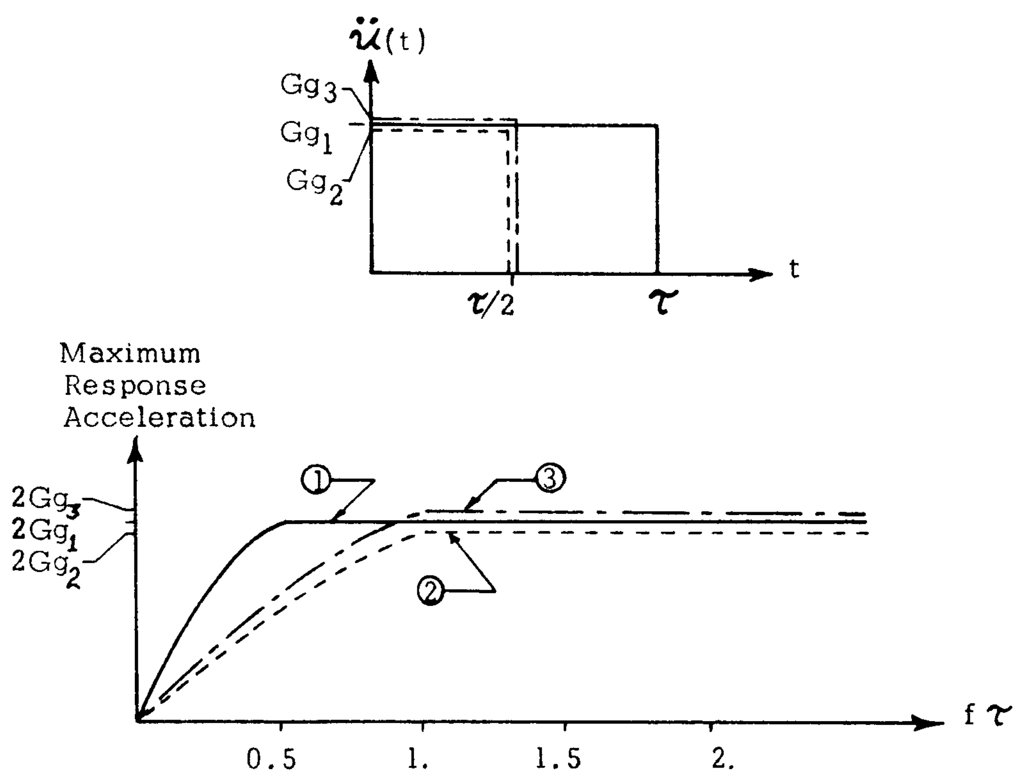


Figure 7 - Comparison of Rectangular Acceleration Pulses and Corresponding Shock Spectra to Illustrate Severity Criteria

Pulse (1) in Figure 7 has an acceleration amplitude G_1 and duration τ which are respectively greater than amplitude G_2 and duration $\tau/2$ for pulse (2). It is evident upon inspection that pulse (1) is greater in all respects than pulse (2) and hence is more damaging. This is confirmed by the fact that shock spectrum (1) is everywhere greater than shock spectrum (2). Similarly, pulse (3) is more damaging than (2).

In comparing pulses (1) and (3) it is not immediately evident which pulse is more damaging based upon a simple inspection of the pulse time-histories. Pulse (3) has a higher acceleration amplitude, but a shorter duration than pulse (1). In this example, comparison of the shock spectra is the only way of establishing the relative damaging potential of the two pulses. Upon examination of the spectra in Figure 7, it can be seen that in the frequency region defined by $f\tau < 1$ shock spectrum (1) is higher than (3). Accordingly, in this frequency range, pulse (1) produces higher responses and must be regarded as more damaging than (3). Similarly, pulse (3) is more damaging than pulse (1) for the frequency region defined by $f\tau > 1$.

4.5 Relative Shock Severity Based on Fourier Spectra

In contrast with the foregoing described simple criterion which is well known for assessing the relative damage potential of shocks through the use of shock spectra, no corresponding criterion has yet been developed for Fourier spectra. The following discussion of this point is drawn from the unpublished notes of References 16 to 18. By definition, Fourier spectra present the continuous distribution of the frequency components contained in a shock motion. Direct visual comparison of Fourier spectra to see which is highest is not valid for assessing relative damaging potential because zero amplitude frequency components can exist at many harmonic frequencies related to the shock duration. In particular, Fourier spectra of symmetrically shaped pulses such as the half-sine, triangular and rectangular pulses, etc., exhibit numerous zeros. The existence of frequencies with zero amplitude in the Fourier spectrum does not preclude the possibility of substantial shock response in systems which are resonant at these frequencies. The shock response of a system is a function of the interaction of all the frequency components in the shock.

To illustrate why direct comparison of Fourier spectra on the basis of amplitude alone is not valid as a damage criterion, consider the comparison of Fourier spectra depicted in Figure 8.

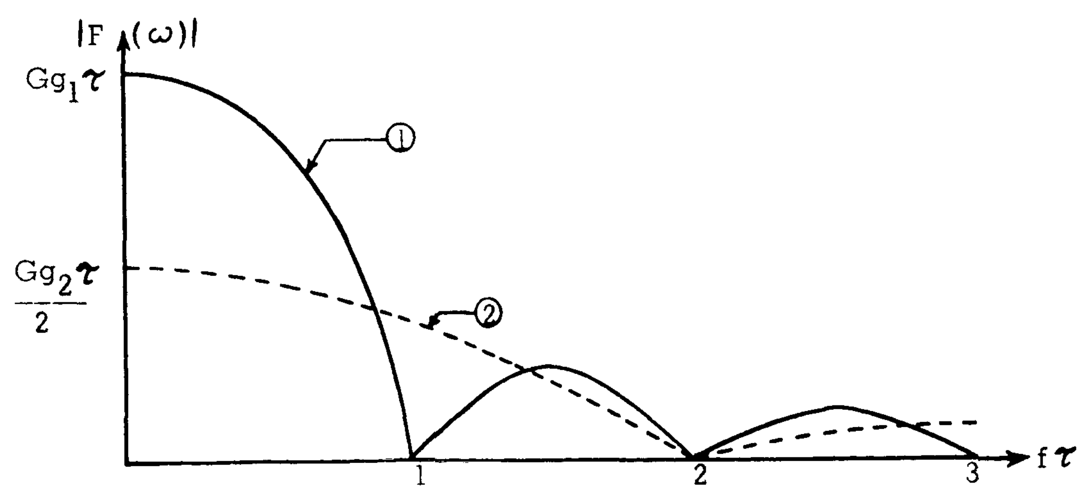


Figure 8 - Fourier Spectra for Rectangular Acceleration Pulses (1) and (2) of Figure 7.

The Fourier spectra in Figure 8 are for rectangular pulses (1) and (2) in Figure 7. Pulse (1) has already been shown in Figure 7 to be more damaging than pulse (2) at all frequencies on the basis of shock spectrum comparisons. Note that if only the Fourier spectrum amplitudes were to be adopted as a criterion of damage, it could be erroniously inferred that pulse (2) is more damaging than pulse (1) in the frequency regions near $f\tau = 1, 3, 5$.

It is evident, therefore, that additional research is necessary, relative to the application of Fourier spectra for comparing damage potential and severity of shocks. This also has a bearing on methods for enveloping the Fourier spectra of shock data discussed earlier.

5. CONCEPTS FOR EXTRAPOLATING SHOCK CONDITIONS

The previous sections have dealt exclusively with the techniques required for analyzing measured service shock conditions and presenting the data in a manner suitable for the development of shock testing criteria. When the available data have been processed, the resulting criteria can only be considered pertinent for the particular vehicle and specific locations represented by the data. Practical limitations in acquiring flight environment data, of course, preclude the possibility of having 100% data coverage for every equipment and component location in a vehicle. Hence, the available data samples must be adjusted or extrapolated to permit establishing shock test criteria at other locations in a vehicle. Further, it is a frequent practical requirement to be able to extrapolate previously developed shock test criteria for new equipment and structural revisions to be made at future times. The theory of mechanical impedance, as described in Reference 19, for example, provides a powerful analytical tool for accomplishing the desired extrapolations.

The mechanical equivalent of Norton's theorem for electrical circuits expresses the change in response characteristics of a system as a function of adding mechanical impedance. For example, consider a structure or foundation of impedance Z_1, which is idealized as comprised of two poles or attachment points as shown in Figure 9. One pole is attached to a driving source from ground, while the other pole is unloaded and exhibits response motion which is conveniently expressed in velocity units. The velocity of the unloaded pole is called the "free velocity" $v_o(t)$ of the structure.

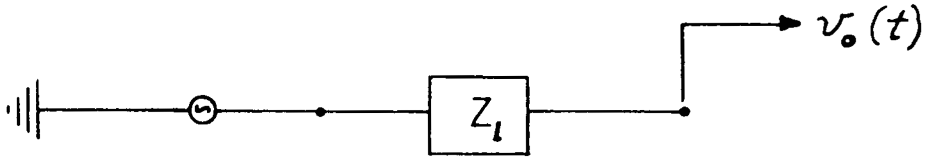


Figure 9 - Impedance Representation of Shock Motion of Unloaded Structure

Now let us consider a second structure or possibly an equipment of impedance Z_2, which is idealized as comprised of two poles or attachment points as shown in Figure 10. One pole is attached to the "foundation" structure and the second pole is considered attached to inertial space, depicted by a "ground" symbol in Figure 10.

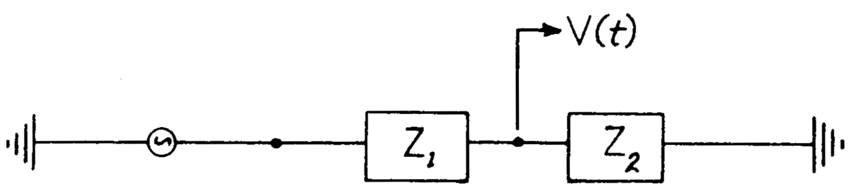


Figure 10 - Impedance Representation of Shock Motion of Loaded Structure

Norton's theorem for adding impedances states that the response motion of the foundation $\underline{v}(\underline{t})$ with the equipment added, can be expressed as a function of the free foundation motion and the impedances of the foundation and equipment as follows:

$$v(t) = V_o(t)\,\frac{Z_1}{Z_1 + Z_2} = V_o(t)\,\frac{1}{1 + Z_2/Z_1} \tag{21}$$

It can be seen from Equation (21) that when the added impedance is negligible compared to the foundation impedance, the shock motion response of the loaded structure will be the same as for the unloaded condition.

The concept of utilizing mechanical impedance to extrapolate shock test criteria is discussed in the following sections.

5.1 Shock Environment Extrapolation Using Fourier Spectra

The transformation depicted by Equation (21) cannot be carried out directly, because in most cases the ratio of impedances on the right hand side of the equation will be a function of frequency. Since many frequency components are involved in shock, it is necessary to carry out the operation indicated by Equation (21) on a spectral basis. Accordingly, the Fourier spectrum equivalent of Equation (21) must be obtained as follows:

$$\left[V(\omega)\right] = \left[V_o(\omega)\right]\frac{Z_1}{Z_1 \quad Z_2} \tag{22}$$

Equations (21) and (22) can only be used if the environmental shock measurements depict "free" or "unloaded" conditions. In the usual case the measured environments represent "loaded" conditions, either due to equipment components in place, or due to series connected structural elements comprising the entire vehicle system. Mechanical impedance theory can be readily used to establish extrapolation relationships for this situation.

Assume in Figure 10 that a new vehicle design will replace the old structure represented by impedance $\underline{Z}_2$ with a new structure $\underline{Z}_3$ at the old interface. The new Fourier spectrum at the old interface becomes:

$$\left[V'(\omega)\right] = \left[V_o(\omega)\right] \frac{Z_1}{Z_1 + Z_3} \tag{23}$$

Combining Equations (22) and (23), an expression is obtained for the new Fourier spectrum as a function of the old spectrum and the revised system impedances, as follows:

$$\left[V'(\omega)\right] = \left[V(\omega)\right] \frac{Z_1 + Z_2}{Z_1 + Z_3} \tag{24}$$

It is recommended that the extrapolation factors represented by the impedance terms in Equation (24) should be further developed to include various values of mass and compliance representative of actual designs.

Then, a family of curves could be established to permit future alterations and adjustments as may be needed in shock test criteria.

5.2 Extrapolation of Shock Spectra

The development of suitable factors for extrapolation of shock environments based on shock spectral curves is not quite so readily accomplished as with Fourier spectra. The reason for this difficulty is that the time-history of an input shock motion cannot be mathematically reconstructed from the shock spectrum curve itself. The input shock time-history is absolutely necessary in order to compute response motion time-histories from which shock spectra are constructed. The process is not reversible. However, the desired shock spectrum extrapolation factors could be developed by a process of comparative correlation studies based upon "loading" of shock time-histories that are initially available. The process would be as follows:

(1) Compute the conventional damped and undamped shock spectra for the given time-history. This represents the initial "unloaded" condition.

(2) Also compute the Fourier spectrum for this "unloaded" condition.

(3) Using Equation (24), establish correction factors based upon a contemplated range of impedance loading situations

and correct the Fourier spectra computed in Step 2. These Fourier spectra now represent various "loaded" conditions.

(4) Using the inverse Fourier transform compute the time-histories for each condition of structure loading represented by the range of impedances selected in Step 3, as follows:

$$v'(t) = \left(\frac{Z_1 + Z_2}{Z_1 + Z_3}\right) \frac{1}{2\pi} \int_{-\infty}^{\infty} V(\omega)\, e^{j\omega t}\, d\omega \qquad (25)$$

(5) Next, damped and undamped responses for the shock time-histories of loaded conditions computed in Step 4 can be computed by the Duhamel integral, which is discussed in greater detail later in this report, as follows:

$$\delta(t) = \frac{1}{\omega_d} \int_0^t \dot{v}'(t)\, e^{-\zeta \omega_d (t-\tau)} \sin \omega_d (t-\tau)\, d\tau \qquad (26)$$

(6) The maxima of the responses computed from Equation (26) in Step 5 represent the shock spectral values of the particular loaded conditions defined by the range of impedances used in Step 3. Direct ratio comparison of loaded and unloaded shock spectra and correlation with the selected impedances should serve to define the desired extrapolation factors for predicting "future" shock spectra based on estimated impedances.

6. COMPUTATION OF SHOCK AND FOURIER SPECTRA

It is presented at length in the previous sections of this report that shock and Fourier spectral analysis is a basic step in all procedures leading to the development of missile component shock test criteria. Methods for computing shock and Fourier spectra are therefore, a prime importance in the present work. Of particular importance are techniques which are pertinent to computing shock and Fourier spectra for highly irregular shock wave forms such as are encountered in actual service environments. There are three methods of practical importance, and these are reviewed briefly in the following sections.

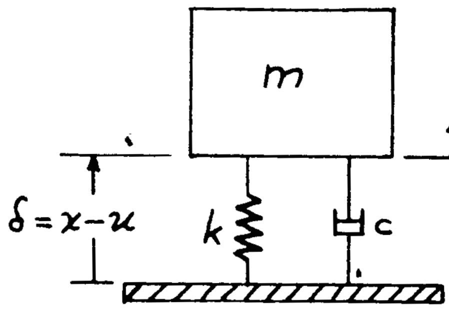
6.1 Analog Method

The use of electronic analog computers comprised of closed-loop circuits with operational amplifiers is well documented in the technical literature for computing damped and undamped shock spectra. See for example, References 8, 13 and 20. The operational analog computer creates a simulation of the damped single-degree-of-freedom system as shown in Figure 11. The equation of motion represented by this simulation is:

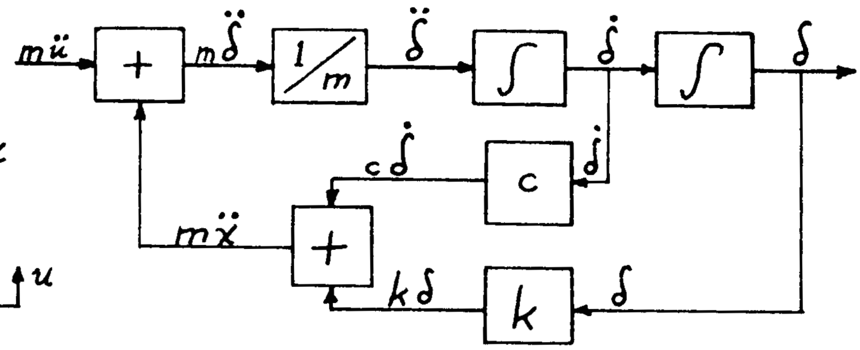
$$m\ddot{x} = c(\dot{u} - \dot{x}) + k(u - x) \tag{27}$$

and in terms of relative motion:

$$m\ddot{\delta} + c\dot{\delta} + k\delta = m\ddot{u} \tag{28}$$



(a) Single Degree-of-Freedom System



(b) Operational Analog Block Diagram Representation of Single Degree-of-Freedom System

Figure 11 - Analog Simulation

Electrical equivalents of highly irregular shock inputs representative of service conditions can be conveniently applied to the analog computer by loops of magnetic tape field data. Alternatively, the "electrical" shock input motion can be created by an electronic function generator such as described in References 8 and 13.

The shock spectrum is obtained by repeating the shock input to the analog to obtain shock response time-histories for progressively varied conditions of system natural frequency and damping. The shock spectrum, as noted earlier and in greater detail in Volume II of this report, is a graph of the maximum system relative deflection responses (times scale factors of 1, ω_n, or ω_n^2), plotted as a function of natural frequency. The Fourier spectrum can be determined from the residual response (after the shock input ends) using the following equation, which is derived in Volume II of this report:

$$\left[\ddot{U}(\omega_n)\right] = \omega_n \left(\delta_r\right)_{MAX} = \left(\dot{\delta}_r\right)_{MAX} \tag{29}$$

6.2 Numerical Method

The analytic solution to Equation (28) which defines the motion of a damped single-degree-of-freedom system which is initially at rest and is subjected to an arbitrary input, is Duhamel's integral as follows:

$$\delta(t) = \frac{1}{\omega_d} \int_0^t \ddot{u}(t)\, e^{-\zeta \omega_d (t-\tau)} \, \mathrm{SIN}\ \omega_d (t-\tau)\, d\tau \tag{30}$$

When the shock input $\ddot{u}(\tau)$ is a highly irregular waveform, then it can be replaced by an equivalent series of rectangular pulses so that the solution of Equation (3) can proceed on a piecemeal basis. The response deflection and velocity at the end of each rectangular pulse represent initial conditions which must be included in the response to each ensuing pulse. The following equations show the addition of initial conditions to Equation (30):

$$\delta(t) = \delta_0 e^{-\zeta\omega_n t}\left(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right) + \frac{\dot{\delta}}{\omega_d} e^{-\zeta\omega_n t}\sin\omega_d t - \frac{1}{\omega_d}\int \ddot{u}(\tau)\, e^{-\zeta\omega_n(t-\tau)}\sin\omega_d(t-\tau)\, d\tau \tag{31}$$

$$\dot{\delta}(t) = -\delta_0\omega_n e^{-\zeta\omega_n t}\sin\omega_d t + \dot{\delta}_0 e^{-\zeta\omega_n t}\left(\cos\omega_d t - \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right) - \int \ddot{u}(\tau)\, e^{-\zeta\omega_n(t-\tau)}\left[\cos\omega_d(t-\tau) - \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d(t-\tau)\right] d\tau \tag{32}$$

It is quite inefficient computationally to use Equation (31) for evaluating the shock response deflection $\delta(t)$ at discrete points in time to an arbitrary input $\ddot{u}(\tau)$ using a digital computer. The integration for every point τ expressed by the last term in Equation (31) must start at the origin or $t = 0$. A more effective digital shock solution technique has been derived in Reference 21 based on a transformation of Equations (31) and (32) into a pair of recursion formulas comprised solely of trigonometric and exponential terms, as follows:

$$\omega_n \delta_{n+1} = A_1\omega_n\delta_n + A_2\dot{\delta}_n + A_3\ddot{u}_n + A_4\Delta\ddot{u}_n \tag{33}$$

$$\dot{\delta}_{n+1} = A_5\omega_n\delta_n + A_6\dot{\delta}_n + A_7\ddot{u}_n + A_8\Delta\ddot{u}_n \tag{34}$$

where

$$\delta_0 = \delta(0), \qquad \dot{\delta}_0 = \dot{\delta}(0)$$

$$A_1 = a + \zeta b, \quad A_2 = b$$

$$A_3 = \frac{1}{\omega_n}(a + \zeta b - 1) = \frac{1}{\omega_n}(A_1 - 1)$$

$$A_4 = \frac{1}{\omega_n}\left\{\frac{1}{\omega_n h}\left[(1-2\zeta^2)b + 2\zeta^2(1-a)\right] - 1\right\}$$

$$A_5 = -b, \quad A_6 = a - \zeta b, \quad A_7 = -\frac{b}{\omega_n}$$

$$A_8 = \frac{1}{\omega_n^2 h}(a + \zeta b - 1) = \frac{A_3}{\omega_n h}$$

$$a = e^{-\zeta\omega_n h}\cos\omega_d h, \quad b = \left(e^{-\zeta\omega_n h}\sin\omega_d h\right)\Big/\sqrt{1-\zeta^2}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}, \qquad \zeta = \frac{c}{2\sqrt{km}}$$

$$\omega_n = \sqrt{\frac{k}{m}}, \qquad h = t_{n+1} - t_n$$

The system response is evaluated with Equations (33) and (34) at equally spaced points $\underline{h}$ in time. The acceleration curve as shown in Figure 12 is replaced with $\underline{m}$ short-time-step acceleration inputs. The solution and its derivitive are first evaluated at time $\underline{t}_n$, i.e., $\delta(\underline{t}_n)$ and $\dot{\delta}(\underline{t}_n)$. Since the differential equation describes a linear system, the solutions $\delta(\underline{t}_n)$ and $\dot{\delta}(\underline{t}_n)$ are a set of initial conditions from which computation of the next values $\delta(\underline{t}_{n+h})$ and $\dot{\delta}(\underline{t}_{n+h})$ can proceed as though the point $\underline{t}_n$ were a new origin translated from $\underline{t} = 0$. A digital computer program based upon Equations (33) and (34) is presented in Volume III of this report. The Fourier spectrum in this program is computed from the residual maximum response using Equation (29).

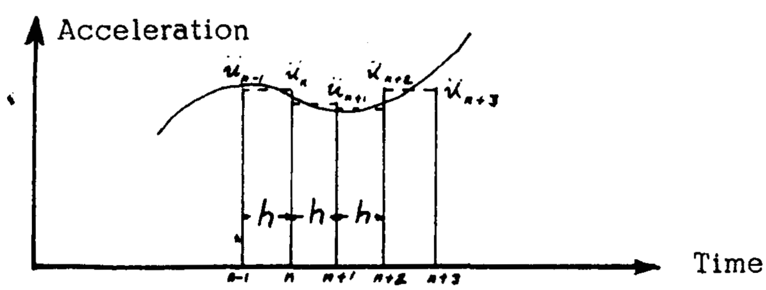


Figure 12 - Portion of an Acceleration Record

6.3 Graphical Method

Several graphical methods are available for the solution of responses of linear systems to transient shock inputs. Of these methods, the phase-plane graphical method described in Reference 22 is the most useful. Like the numerical procedure, the shock motion is replaced by a sequence of rectangular pulses. The response to each pulse is then generated as a vector whose rotational velocity is ω_n and whose "horizontal projection" is proportional to velocity and "vertical projection" is proportional to displacement. The plane of the rotating vector is called the "phase-plane". The maximum value of the response for each value of ω_n determines one point on the shock spectrum.

7. INVESTIGATION OF PUBLISHED STAGE SEPARATION SHOCK DATA

During the course of the present program, a brief study was made of available published data on shocks representative of stage separation conditions produced by pyrotechnic devices. Samples of data for this study were obtained from References 23 to 26 and are presented in Figures 13 to 16, together with acknowledgement as to the source. The data are presented as shock time-histories and/or shock spectra, as originally published. To facilitate comparison the published two-coordinate spectra of Figures 13 to 16 were replotted in four-coordinate format as shown in Figure 17.

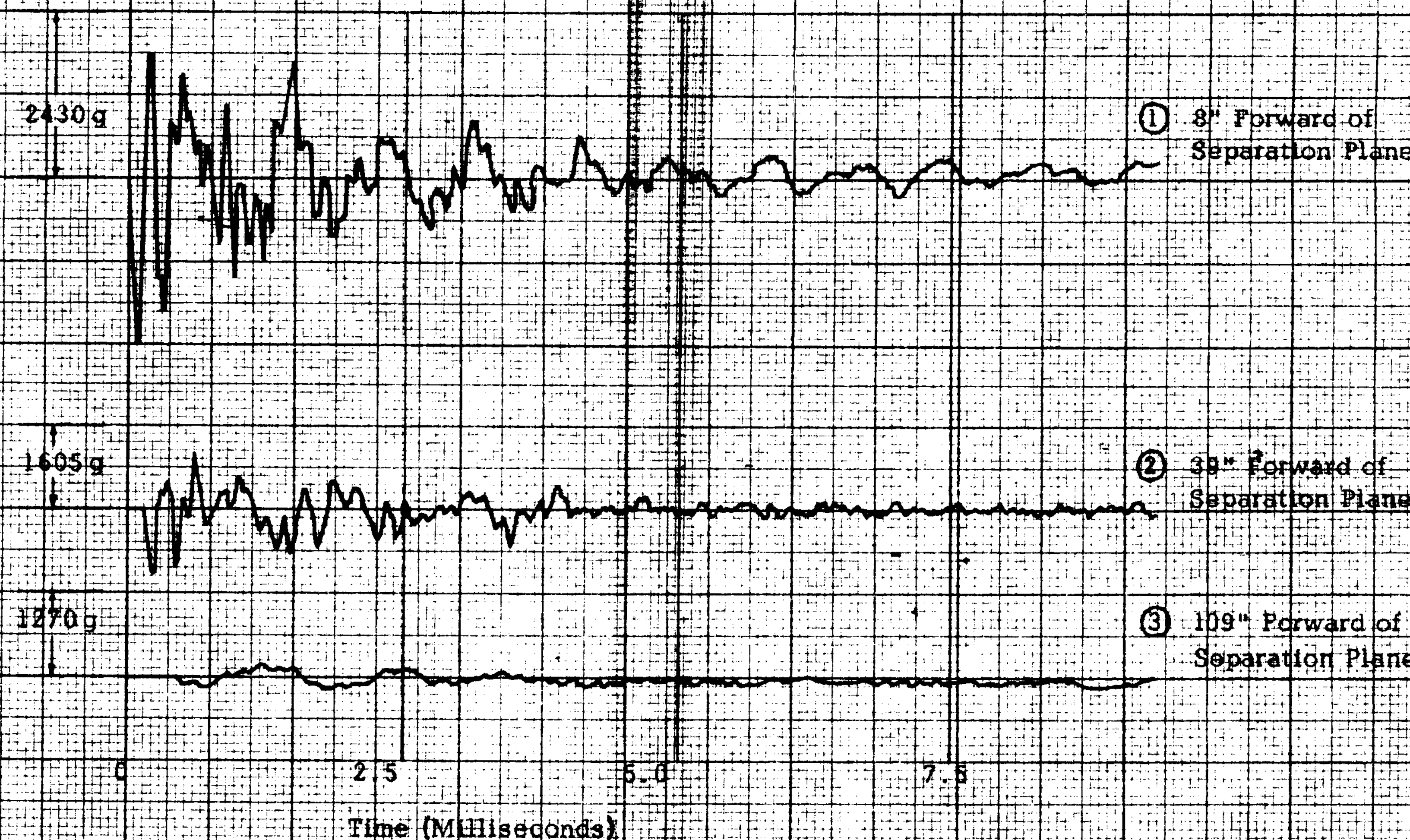


FIGURE 13: Pyrotechnic Shock Measured Along A Stringer

After Roberge and Rybacki (Ref. 23)

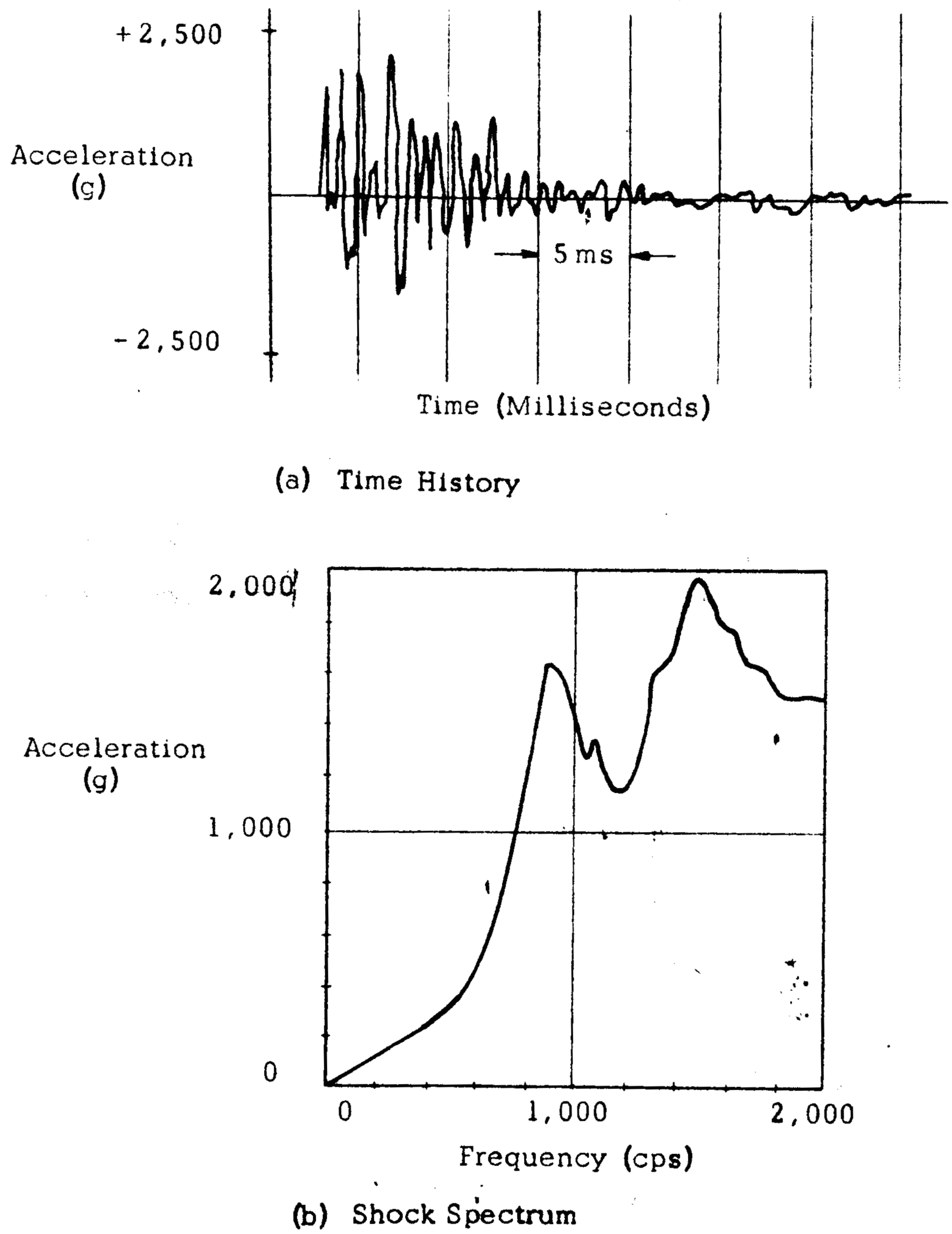
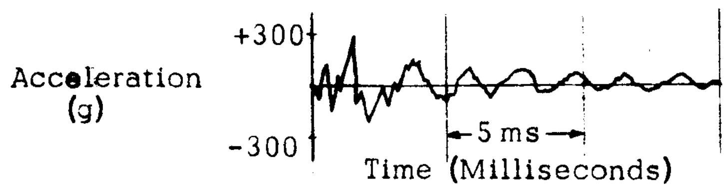
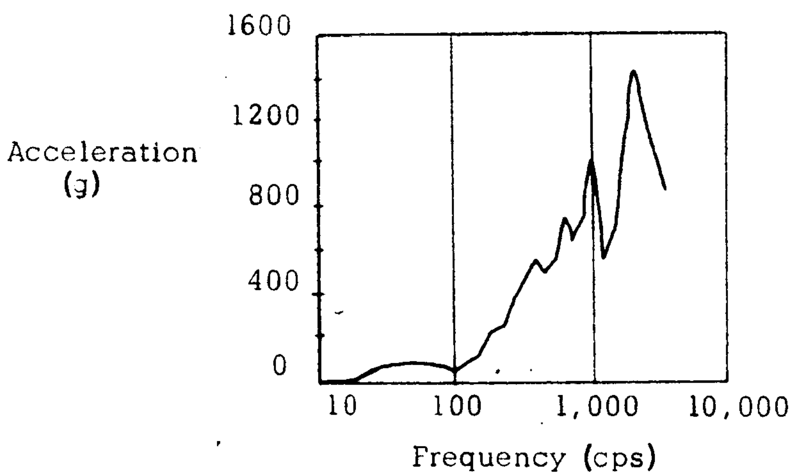


FIGURE 14 Pyrotechnic Shock Data From Lockheed

After Paul (Ref. 24)



(a) Time History



(b) Shock Spectrum

FIGURE 15 Pyrotechnic Shock Data From Douglas

After Hines (Ref. 25)

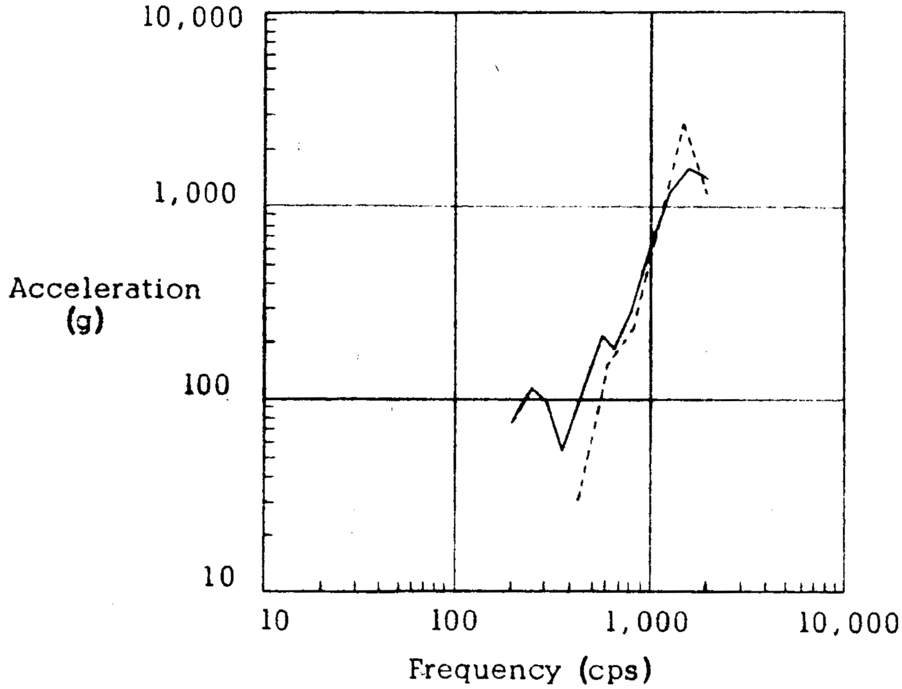


FIGURE 16: Pyrotechnic Shock Spectra From Lockheed

After Blake (Ref. 26)

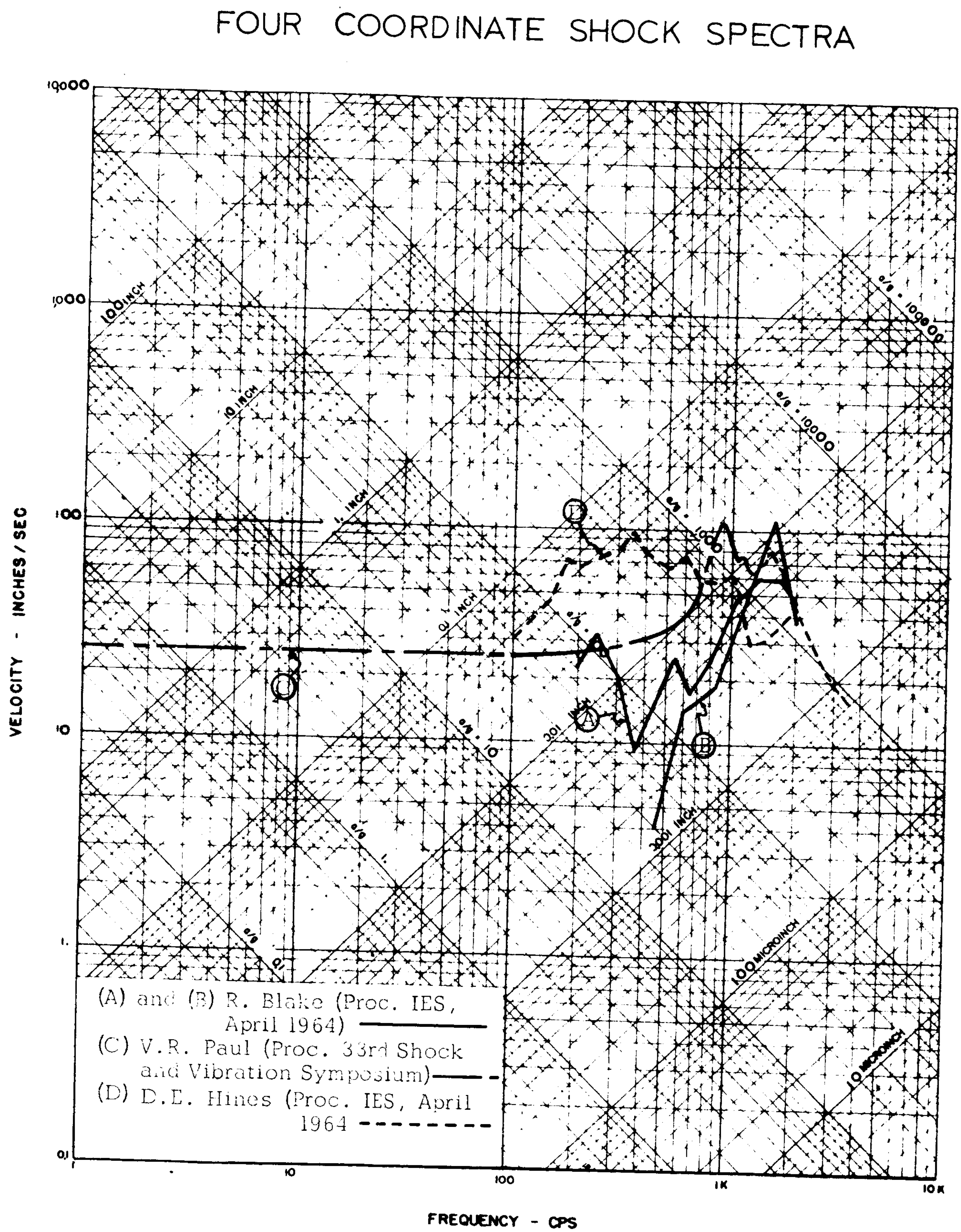


FIGURE 17: Typical Separation Shock Spectra

With the exception of spectrum "C" in Figure 17, the available spectra provide no low frequency information. This is unfortunate because low frequency shock spectrum responses are "impulse sensitive" and serve to define the velocity change of the shock input. This feature of low frequency response data is only illustrated by spectrum "C". The original data for spectrum "D" in Figure 17 was reported down to 10 cps as shown in Figure 15(b). However, it can be seen in Figure 15(a) that the spectrum is based on a complex transient whose duration is arbitrarily cut off at 15 milliseconds. Accordingly, it was deemed appropriate to discount spectrum frequencies responsive to this duration. Namely, the spectrum values below 100 cps were omitted in plotting spectrum "D" in Figure 17.

For purposes of comparison of service measured shock conditions with idealized forms, Figure 18 shows a superposition of shock spectra for a single frequency decaying sinusoid, a half-sine pulse, and separation shock pulse "C" from Figure 17. Spectra for other idealized pulses appear in Figure 5 and Volume II of this report. The acceleration and duration of the half-sine pulse and decaying sinusoid pulse have been adjusted to provide a maximum response acceleration of 2000g at 1600 cps to match the separation shock. For purposes of simplicity, no attempt was made to match the 900 cps component in the separation shock.

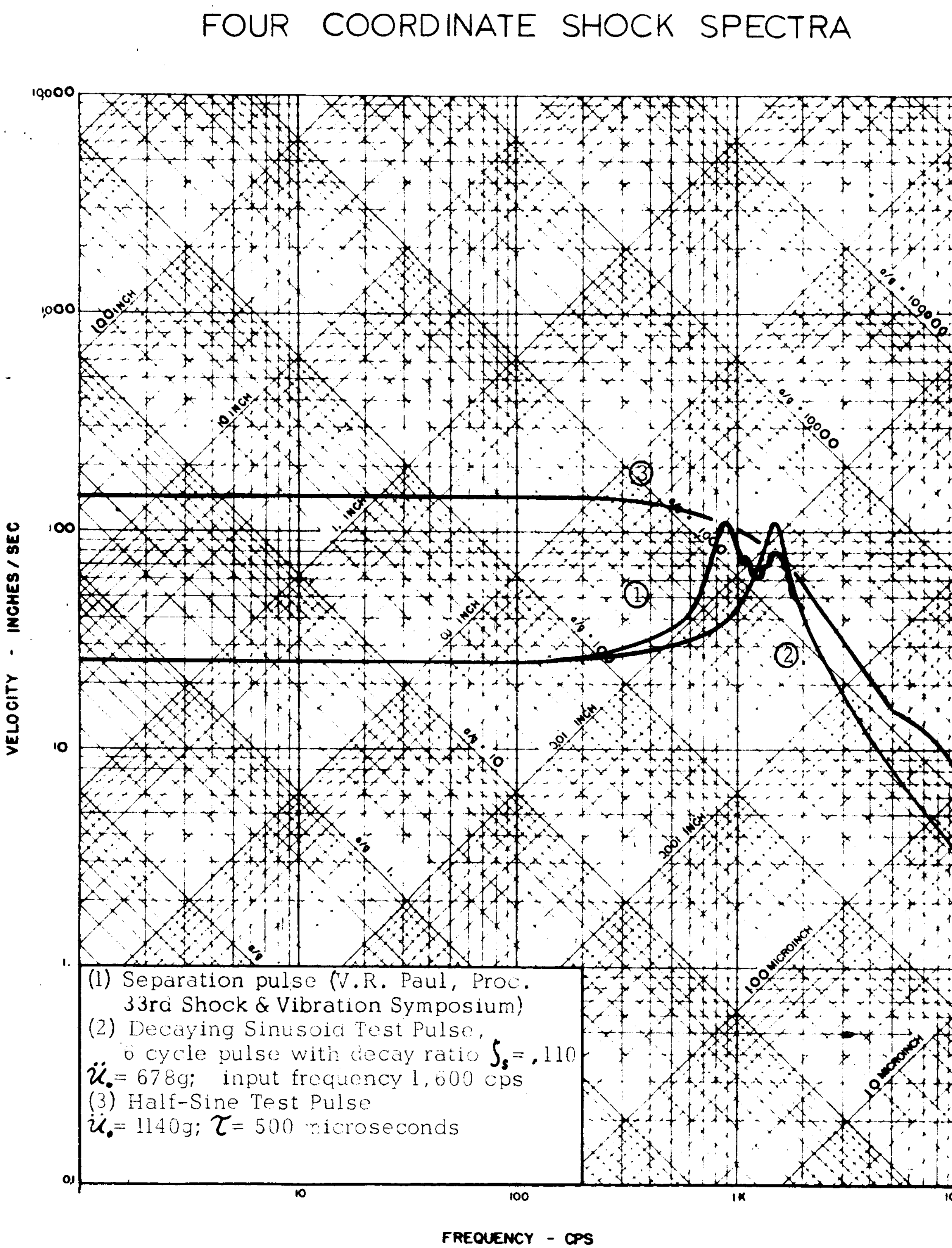


FIGURE 18: Separation Shock Test Pulse Comparison

It is significant to note in Figure 18 that the decaying sinusoidal transient can be adjusted to provide a reasonable match with the peak system responses induced by a separation shock in both low and high frequency ranges. The half-sine pulse (or other similar simple pulses) can only be adjusted for either a high frequency or a low frequency match, but not both. It can be tentatively concluded that in so far as separation shock pulses are concerned, simple shock pulses are far from ideal for missile and rocket component testing. This should be a subject of further detailed study.

8. SUMMARY

The three volumes comprising this final report present a comprehensive review and discussion of available theoretical and empirical concepts leading to the derivation of missile component shock specifications. Volume I presents and discusses the merits and disadvantages of various methods for specifying measured shock conditions and extrapolating the data to represent new vehicles or component installations. Volume II presents a comprehensive compilation of shock and Fourier spectra for idealized shock pulse shapes which either have been or could be used for laboratory shock testing purposes. These spectra have been normalized and are plotted in four coordinate (logarithmic) format to facilitate correlation with other data. It is suggested that the selection of a particular idealized laboratory shock test can be based upon spectra from Volume II that compare closely with spectra derived from service data in accordance with the concepts of Volume I. A digital computer program for obtaining shock and Fourier spectra for any arbitrary shock motion is presented in Volume III.

9. RECOMMENDATIONS FOR FUTURE WORK

It is recommended that the concepts developed herein for the derivation and extrapolation of missile component shock tests should be further refined and reduced to practice. Particular questions which arose during the course of the present work are as follows:

(1) Do the frequency "notches" which occur in Fourier spectra for symmetrical pulses have any significance in comparing the relative severity of shock motions?

(2) Can the above noted frequency "notches" be ignored in enveloping Fourier spectra?

(3) What is the optimum length of record to analyze in an arbitrary or complex shock time-history?

(4) How closely do existing shock machine spectra compare with the idealized pulse spectra in Volume II?

(5) Establish correction factors based on mechanical impedance theory for extrapolation of specifications based on equipment and/or vehicle structure modifications.

(6) Compute shock and Fourier spectra for actual measured flight shock data and correlate with the idealized spectra in Volume II. This includes analysis of pyrotechnic shock data measured in the laboratory and in flight.

The end objective of the work outlined above should be to develop an interim shock specification for immediate use in qualifying missile components. A longer range objective would be to establish the simplest method for developing missile component shock tests based on actual environments and present this in a handbook format for application to future missiles.

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22. Jacobsen, L.S., and Ayre, R.S., "Engineering Vibrations". McGraw-Hill Book Co., Inc., New York, 1958.

23. Roberge, H.J. and Rybacki, J., "Shock Environments Generated by Pyrotechnic Devices". Bulletin 33, Shock, Vibration and Associated Environments. Office of the Director of Defense, March 1964.

24. Paul, V.R., "Mechanical Shock from Frangible Joints". Bulletin 33, Shock, Vibration and Associated Environments. Office of the Director of Defense, March 1964.

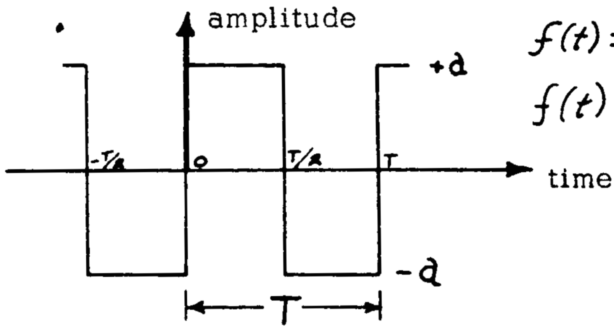
25. Hines, D.C., "Generation and Propagation of Stage Separation Shocks in Missiles and Space Vehicles". 1964 Proceedings of the Institute of Environmental Sciences, April 1964.

26. Blake, R.E., "Problems of Simulating High-Frequency Mechanical Shocks". 1964 Proceedings of the Institute of Environmental Sciences, April 1964.

APPENDIX A

Examples of Fourier Series and Fourier Integral Spectral Analysis of Simple Analytic Waveforms

To illustrate the application of Fourier series to define a periodic function, consider the repeating square wave shown in Figure A-1.



$$f(t) = -a \qquad -T/2 < t < 0$$
$$f(t) = +a \qquad 0 < t < T/2$$

Figure A-1 - Periodic Square Wave

Using the exponential form of Equation (6), an expression for the Fourier coefficients of the periodic square wave in Figure A-1 may be determined as a function of n, as follows:

$$C_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-jn\omega t}\, dt = \frac{1}{T}\int_{-T/2}^{0} -a\, e^{-jn\omega t}\, dt + \frac{1}{T}\int_{0}^{T/2} a\, e^{-jn\omega t}\, dt$$
$$= \frac{a}{T}\left[\frac{-1}{-jn\omega}\, e^{-jn\omega t}\right]_{-T/2}^{0} + \frac{a}{T}\left[\frac{1}{-jn\omega}\, e^{-jn\omega t}\right]_{0}^{T/2}$$
$$= \frac{a}{-jn\omega T}\left[-1 - e^{jn\omega T/2} - e^{-jn\omega T/2} - 1\right] = \frac{a}{jn\omega T}\left[2 - 2\cos\frac{n\omega T}{2}\right]$$

Substituting fundamental frequency $\omega = \frac{2\pi}{T}$

$$C_n = j\,\frac{a}{n\pi}\left(\cos n\pi - 1\right)$$

The calculation of the Fourier coefficients and phase angles for the periodic square wave is summarized in Table A-1 and the point or line spectra plotted in Figure A-2.

Table A-1 Summary of Fourier Coefficient and Phase Angle Calculation for Periodic Square Wave

n	$\lvert c_n \rvert$	θ_n
0	0	---
1	$2a/\pi$	$-j = -90^{\circ}$
2	0	---
3	$2a/3\pi$	$-j = -90^{\circ}$
4	0	---
5	$2a/5\pi$	$-j = -90^{\circ}$
6	0	---
7	$2a/7\pi$	$-j = -90^{\circ}$

The Fourier series for the periodic square wave is from Equation (7):

$$\begin{aligned} f(t) &= C_0 + 2\sum Re\left(C_n e^{jn\omega t}\right) \\ &= 0 + \sum 2C_n\, Re\,(\cos n\omega t + j \sin n\omega t) \\ &= \frac{4a}{\pi}\left(\sin \omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \cdots\right) \qquad \text{(A-1)} \end{aligned}$$

It may be noted that Equation (A-1) contains only sine terms. This result occurs for all "odd functions", i.e., $f(t) = -f(-t)$. If the function is an "even function", i.e., $f(t) = f(-t)$. The function is a mirror image about $t = 0$ and the series will consist of cosine terms only.

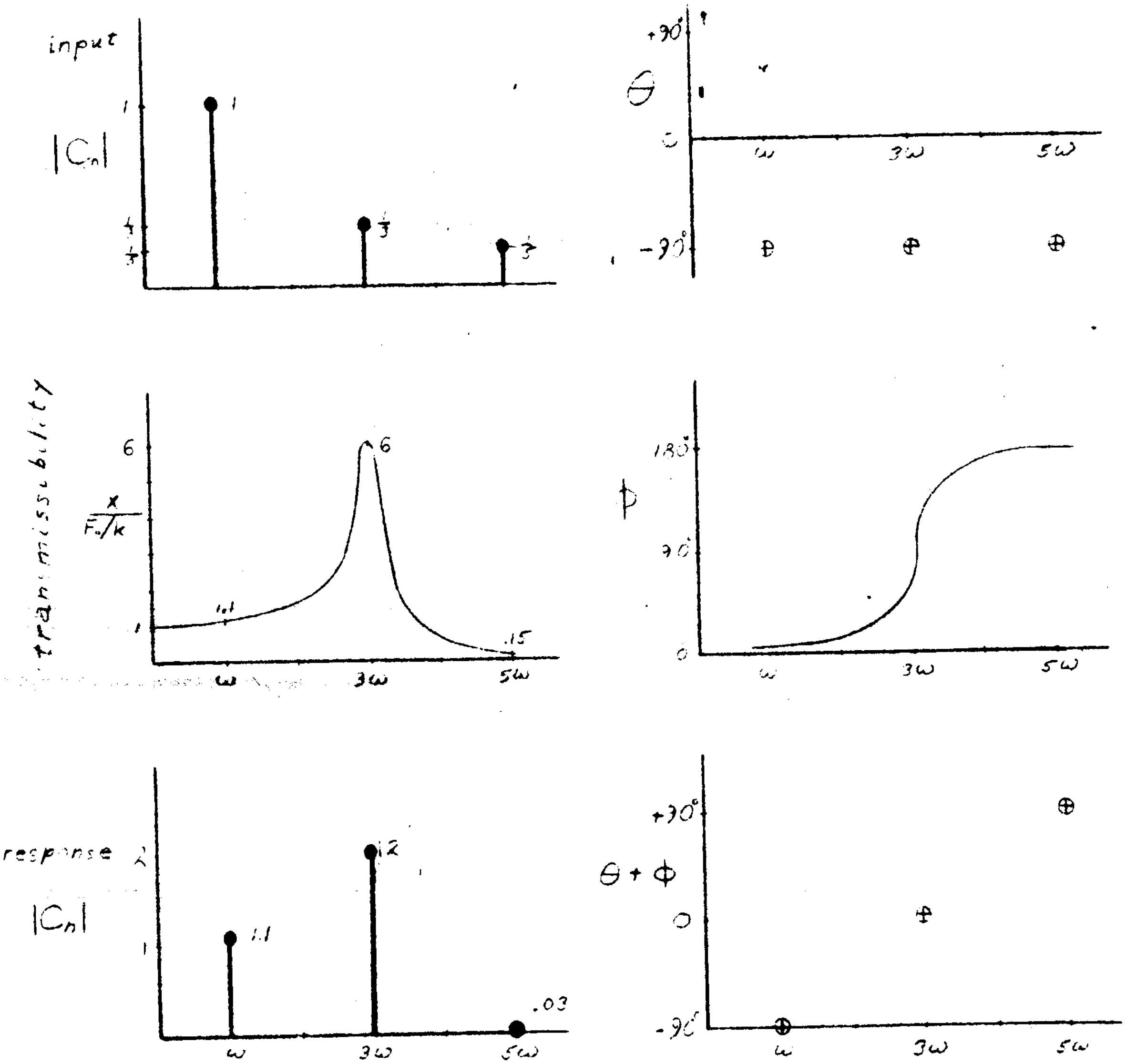
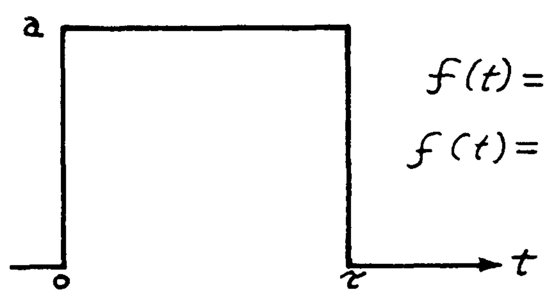


Figure A-2 Illustration of Input and Response of Fourier Coefficient and Phase Angle Line Spectra for Periodic Square Wave Applied to Simple System with Q = 6.

The response of a linear system to a complex periodic input will also be a complex periodic motion. The Fourier coefficients defining the amplitudes of the harmonic components in the response are obtained by multiplying each component of the input by the transmissibility of the system at the frequency of each component. The system response phase angle is simply added to the initial phase angle of the component to establish the final phase angle relationships between all the response components. The Fourier components and phase angles of the response of a lightly damped single-degree-of-freedom system to the periodic square wave in the previous example are illustrated in Figure A-2.

To illustrate the application of Fourier integral to define a transient function, consider the rectangular pulse shown in Figure A-3.



$f(t) = a \qquad 0 \le t \le \tau$

$f(t) = 0 \qquad t < 0 \text{ and } t > \tau$

Figure A-3 - Rectangular Pulse

Using Equation (10), a generalized expression for the Fourier transform of the rectangular pulse in Figure A-3 is obtained as follows:

$$F(\omega) = \int_{-\infty}^{\infty} a\, e^{-j\omega t}\, dt = \int_{0}^{\tau} a\, e^{-j\omega t}\, dt = \frac{a}{-j\omega}\left[e^{-j\omega t}\right]_0^{\tau}$$

$$= \frac{a}{-j\omega}\left(e^{-j\omega t} - 1\right) = \frac{a}{-j\omega}\left(\cos \omega t - j \sin \omega t - 1\right)$$

$$= \frac{a}{\omega}\left[\sin \omega t - j\,(1 - \cos \omega t)\right] \qquad \text{(A-2)}$$

The absolute value of Equation (A-2) for the Fourier transform of a rectangular pulse is obtained by substitution in Equation (15) as follows:

$$|F(\omega)| = \frac{a}{\omega}\sqrt{\sin^2 \omega\tau + 1 - 2\cos \omega t + \cos^2 \omega\tau}$$

$$= \frac{2a}{\omega}\sqrt{\frac{1 \ \ \cos \omega\tau}{2}} = \frac{2a}{\omega}\left|\sin \frac{\omega\tau}{2}\right|$$

$$= \frac{\tau a}{\omega\tau/2}\left|\sin \frac{\omega\tau}{2}\right| \qquad \text{(A-3)}$$

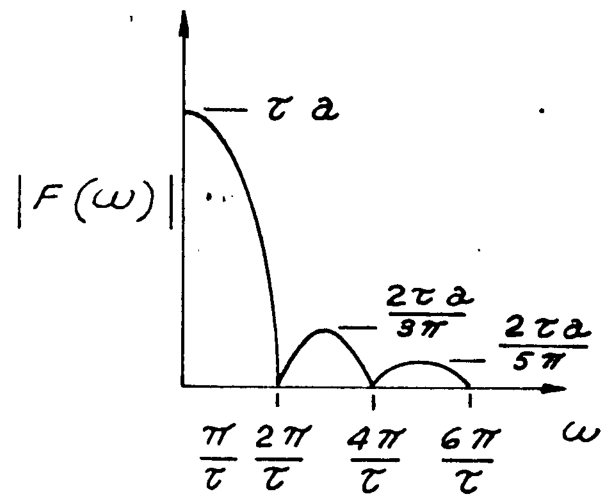
The phase angle of the transform is from Equation (16):

$$\tan \theta(\omega) = \frac{Im\left[F(\omega)\right]}{Re\left[F(\omega)\right]} = \frac{1 - \cos \omega t}{\sin \omega t} = \tan \frac{\omega\tau}{2}$$

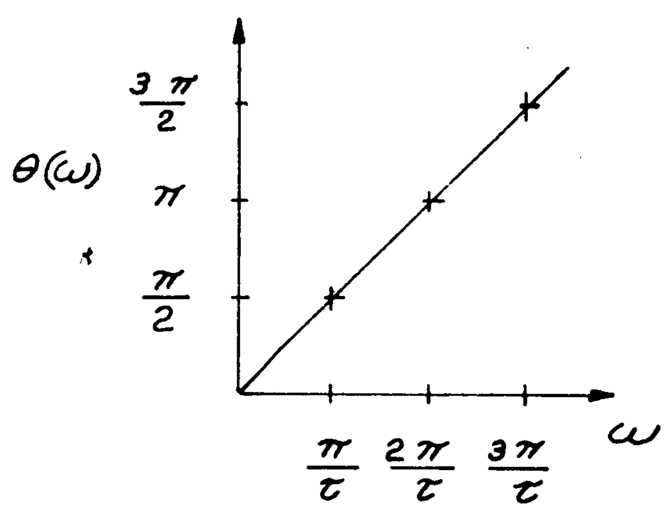
$$\theta(\omega) = \frac{\omega\tau}{2} \qquad \text{(A-4)}$$

Table A-2 Calculation of Fourier Integral Spectrum and Phase Angle for Rectangular Pulse

(ω)	$\omega\tau/2$	$\sin(\omega\tau/2)$	$\lvert F(\omega)\rvert$
0	0	0	$\tau a(1)$
π/τ	$\pi/2$	1	$\tau a(2/\pi)$
$2\pi/\tau$	π	0	0
$3\pi/\tau$	$3\pi/2$	1	$\tau a(2/3\pi)$
$4\pi/\tau$	2π	0	0



(a) Fourier Frequency Spectrum



(b) Fourier Phase Angle Spectrum

Figure A-4 - Fourier Frequency and Phase Angle Spectra for Rectangular Pulse

Plots of the Fourier integral (frequency) spectrum and phase angle spectrum from Table A-2 calculations for the rectangular pulse of Figure A-3, are shown in Figure A-4. The Fourier frequency and phase angle spectra of a system response are obtained in the same manner as indicated in Figure A-2 for the periodic function.

APPENDIX B

Brief Bibliography of Technical Literature Pertinent to Shock Environments, Testing and Analysis

The bibliography presented herein is divided into four sections as follows:

B-1 Shock Testing Machine and Methods.

B-2 Shock Environment.

B-3 Shock Spectrum, Fourier Spectrum and Analysis.

B-4 Impedance.

B-1 Shock Testing Machine and Methods

Apgar, J.W. and Thomson, G.R., "The Reverse-Action Shock-Testing Method", Bulletin 28, Shock, Vibration and Associated Environments, Department of Defense, August 1960.

ABSTRACT

A versatile shock-testing method has been developed in which a deflected structural member, such as a cantilever, is suddenly released. At the bottom of the second quarter cycle, the object being tested is disconnected. Selection of pulse duration is flexible, and the device can be designed for testing large items.

Blake, R.E., "Problems of Simulating High-Frequency Mechanical Shocks". 1964 Proc. IES, April 1964.

ABSTRACT

Thoughts are presented about high frequency motion (e.g. 500 to 2000 cps) with a characteristic of high acceleration to displacement ratio. Shock spectra of explosive joints are compared with the spectra of a half-sine pulse.

Brooks, R.O., "Shock Test Methods Versus Shock Test Specifications". Bulletin 31, Shock, Vibration and Associated Environments. Department of Defense, March 1963.

ABSTRACT

Current shock test shortcomings are discussed and some techniques are presented to aid the designer and test engineer in evaluating component shock test data.

Crede, Charles E., "Fundamentals of Shock Testing". 1963 Proc. IES, April 1963.

ABSTRACT

A discussion of the basic philosophy of shock testing with an introduction to test requirements and difficulties in performing tests. A means of specifying a shock test is presented.

Crede, Charles and Junger, Miguel, "A Guide for Design of Shock Resistant Naval Equipment", Bureau of Ships, NAVSHIPS 250-660-30, Washington, D. C., 1949.

ABSTRACT

It is the objective of this publication to assist designers, particularly those who are not experienced in the design of Naval equipment, by informing them of methods for design purposes. It is hoped that the number of cut-and-try steps necessary to arrive at a final design will be reduced. An attempt is made to provide a correlation between the analytical approach to the problem and the empirical methods which have been used in the past for designing shock resistant equipment.

Dinicola, D.J., "A Method of Producing High-Intensity Shock with an Electro-Dynamic Exciter". 1964 Proc. IES, April 1964.

ABSTRACT

This paper reports on the feasibility of employing an electro-dynamic exciter with a light-weight armature and high-force capability to generate high-level, saw-tooth shock pulses lasting for 5 msec or less with amplitudes up to 170 G peak. This system appears to be valid and dependable for testing specimens up to approximately 30 lbs.

Eubanks, R.A. and Juskie, Bernard R., "Shock Hardening of Equipment", Bulletin 32, Shock, Vibration and Associated Environments. Department of Defense, December 1963.

ABSTRACT

Extensive study of the results of shock tests provides a procedure which can be used to select equipment for shock survival. A shock spectrum is evaluated mathematically and compared to characteristics of shock testing facilities.

Gwinn, Jr., J.T.,"Suspension System Design to Reduce High Intensity Shock", Bulletin 31, Shock, Vibration and Associated Environments. Department of Defense, March 1963.

ABSTRACT

This paper presents basic concepts that must be recognized and used in the design of resilient suspensions to attenuate high intensity shock. The theoretical response of a given system may be calculated from measured test data for the Navy Hi-Impact Shock Testing Machine and known fragility data for the equipment to be protected. Such a design problem is reviewed. Actual test data taken during shock testing of the system is compared to the calculated response and proves the design approach.

Hasslacher III, George J., "Development of Method, Equipment, Instrumentation and Techniques for Determining the Inherent Fragility Rating of Military Material, Technical Report No. 11". Pennsylvania State University, Army Contract No. DA19-129-QM-804, ASTIA AD235149, August 1959.

ABSTRACT

A study of the characteristics of the HYGE shock tester was made. The effect of thrust rod load, firing pressure, oil level, acceleration pin shape and deceleration pin size was studied. Curves showing the effect of those parameters on the shock pulse are presented. A study of the use of the HYGE with a carriage system is also presented. Recommendations for further studies are included.

Hasslacher III, George J., "Development of Methods, Equipment, Instrumentation and Technique for Determining the Inherent Fragility Rating of Military Material, Technical Report No. 12". Pennsylvania State University, Army Contract No. DA19-129-QM-804, ASTIA AD235148, August 1959.

ABSTRACT

A method of using the impulse response of a simple structure to predict the structures response to an arbitrary shock pulse is presented. The prediction of impulse response from sinusoidal response is presented. The computation of shock spectra using impulse techniques is shown. Experimental verification of the results is given.

Lewis, H.O., "Shock Testing Polaris Missile Re-Entry Bodies With an Electrodynamic Shaker", Bulletin 28, Shock, Vibration and Associated Environments. Department of Defense, August 1960.

ABSTRACT

This paper describes a method devised for shock testing Polaris re-entry bodies wherein any available electrodynamic shaker of 25,000 pounds force output can be used. The versatility of the system is emphasized in that the shock pulse can be modified within wide limits and may be superimposed on vibration.

Lonborg, James O., "A Slingshot Shock Tester". 1963 Proc. IES, April 1963.

ABSTRACT

This paper describes the design, development and operation of a shock tester capable of subjecting small specimens to accelerations up to approximately 40,000 g with impact velocities to 200 fps. The test specimen is mounted on a rail-guided carriage. Elastic shock-absorber cord (bunges) is used to impact the desired impact velocity to the specimen carriage. The test shock is produced by controlled impact between the carriage nose and a plastically deformed target.

Lunny, E.J. and Gertel, Maurice, "Vibration, Shock and Acceleration Testing", Space Aeronautics, October 1962.

ABSTRACT

The fundamental concepts of vibration, shock and acceleration testing are presented. Vibration test curves are illustrated for equipment in air and ground-launched missiles. A Table of Mil-Spec. shock tests is shown with the following information: specification, simulated condition, nature of shock and commonly used machine. A method of developing shock test procedures is proposed.

Mains, R.M., "Introduction to Shock and Vibration Simulation", Bulletin 28, Shock, Vibration and Associated Environments. Department of Defense, August 1960.

ABSTRACT

Shock and vibration testing or simulation are essential for demonstrating what improvements can be or have been made in the design, for determining the adequacy and acceptability of the design, and for controlling the quality of the product. These various functions of testing or simulation are discussed in this paper.

McWhirter, M., "Shock Machines and Shock Test Specifications", 1963 Proc. IES, April 1963.

ABSTRACT

The various types of shock machines are discussed along with a detailed discussion on obtaining shock pulses and accomplishing certain tests. The information that must be included in a good test specification, such as duration, pulse shape and rise time are discussed.

Meagher, Thomas F., "A High-Energy Shock Facility Using Electromagnetic Energy". 1964 Proc. IES, April 1964.

ABSTRACT

The repulsion-coil technique for energy conversion offers a powerful, new tool in the field of acceleration testing. Using this technique, a prototype magnetic shock device was developed that uses electrical energy stored in a capacitor bank to produce high-g, mechanical acceleration pulses.

This investigation is intended to demonstrate the application of electrical-energy conversion in the production of a controllable acceleration pulse, toward development of a magnetic shock device for component testing and instrument calibration.

Melcher, Stanley R., "The Air Shock Tube as a Shock Testing Facility". Bulletin 28, Shock, Vibration and Associated Environments. Department of Defense, August 1960.

ABSTRACT

At the Naval Missile Center a shock tube has been used to generate square wave shock pulses for shock testing of components. This paper describes the facility, typical test configurations, and results.

Painter, G.W. and Parry, H.J., "Simulating Flight Environment Shock on an Electrodynamic Shaker", Bulletin 33, Shock, Vibration and Associated Environments. Department of Defense, March 1964.

ABSTRACT

A program to develop a method to produce oscillatory acceleration transients with adjustable shock spectra will be described. Using this method, a wide range of pulse conditions can be accommodated including an "average" or "data envelope" type pulse. The equipment used and some results of the experimental program to date will be discussed.

Sivinski, H.D., "Acceleration-Time Pulse Shaping with an Air-Gun Facility", Bulletin 29, Shock, Vibration and Associated Environments. Department of Defense, June 1961.

ABSTRACT

A 5.5-inch-diameter air-gun facility was designed and built for component shock testing. It can produce a variety of commonly required shock-pulse shapes of velocity change levels far above those naturally available in other types of laboratory shock-test facilities. The pulses are very clear and repeatable.

Thorne, L.F., "The Design and Advantages of an Air-Accelerated Impact Mechanical Shock Machine". Bulletin 33, Shock, Vibration and Associated Environments. Department of Defense, March 1964.

ABSTRACT

Modifications were made to a standard 6 inch HYGE mechanical shock actuator, so that "reverse firing" could be accomplished. In this application, the HYGE is utilized as a velocity generator by accelerating a shaft-mounted specimen carriage into an impact material positioned between the carriage and the top of the HYGE machine.

Vigness, I. and Clements, E.W., "Sawtooth and Half-Sine Shock Impulses From the Navy Shock Machine for Medium Weight Equipment". Report 5943, June 1963.

ABSTRACTS

A shock test is frequently specified as an acceleration pulse of controlled shape and magnitude and which has a desired shock spectrum. A description is given as to how the Navy High-Impact Shock Machine for Medium-weight Equipment can be made to provide acceleration pulses of a half-sine or sawtooth shape by inserting elastic or plastic materials between various impacting surfaces. Details as to the shape of the inserts are given for nominal 6-millisecond sawtooth pulses having amplitudes up to 60 g, and for relatively long (10-20 milliseconds) low-amplitude half-sine pulses. A maximum velocity-change up to 8 ft/sec is possible. The acceleration amplitudes involved in stopping the anvil are generally small compared with the initial pulse. Displacements are limited to 3 inches.

Weiner, R.S., "Deduction of Shock Testing Methods and Facilities Criteria". Defense Atomic Support Agency Report DASA 1328, ASTIA AD294177, December 1962.

ABSTRACT

Criteria are deduced for shock-testing methods and facilities to test components which must sustain the severe mechanical shocks of nuclear environments.
Longitudinal and transverse stress pulses propagating through elementary structural members are studied to provide insight to the more complicated problems of pulses propagating through a general piece of equipment.
The effect of higher-frequency oscillations which accompany the nuclear environment is investigated by comparing the response of spring-mass systems subjected to similar pulses without the higher frequencies.

Wells, R.H. and Mauer, R.C., "Shock Testing with the Electrodynamic Shaker", Bulletin 29, Shock, Vibration and Associated Environments. Department of Defense, June 1961.

ABSTRACT

A growing interest in the use of electrodynamic shakers to perform shock tests has led the authors to the preparation of this paper. In it are an outline of experience obtained over the past five years and a summary of the current state-of-the-art at the Lockheed Aircraft Corporation, California Division.

B-2 Shock Environment

Hines, D.C., "Generation and Propagation of Stage Separation Shocks in Missiles and Space Vehicles", 1964 Proc. IES, April 1964.

ABSTRACT

Measured data on vehicle stage separation is discussed. Explosive bolts, linear shaped charge and "soft" joint separation are considered. These mechanisms are compared as to cost, reliability, and severity of shocks produced in order to aid designers in their choice of mechanisms. Data acquisition and data reduction techniques are discussed. Scaling laws, based on the limited experimental data and analyses are presented.

Lowe, Russell and Cavanaugh, Richard D., "Correlation of Shock Spectra and Pulse Shape with Shock Environment". J. Inst. Environmental Eng., Vol. 1, No. 1, February 1959.

ABSTRACT

This paper presents a comprehensive examination of the "Shock Spectrum Philosophy" and discusses the following topics. The ramifications of sawtooth shock spectra and sawtooth pulse simulation. The need for standardization of the shock pulse as well as the response spectra. The area of usefulness of sawtooth shock pulse testing. Concluding with the areas where sawtooth shock testing is not necessary.

Paul, V.R., "Mechanical Shock from Frangible Joints". Bulletin 33, Shock, Vibration and Associated Environments. Department of Defense, March 1964.

ABSTRACT

This paper describes the work being done on the frangible joint shock problem at LMSC and gives the results obtained to date. The shocks produced from frangible joints during stage separation are of extremely high amplitude (100 to 3000 g) at high frequencies (600 to 6000 cps).

Roberge, H.J. and Rybacki, J., "Shock Environments Generated by Pyrotechnic Devices", Bulletin 33, Shock, Vibration and Associated Environments. Department of Defense, March 1964.

ABSTRACT

A testing program to study shocks generated by pyrotechnic separation devices is described and typical shock response data is presented. Inadequacies of existing instrumentation, simulation, and analysis techniques are discussed. Remedial measures that were employed to negate the shock effects are described.

B-3 Shock Spectrum, Fourier Spectrum and Analysis

Atchison, Mrs. S.C., "Shock Data Handling Systems at David Taylor Model Basin," Bulletin 28, Shock, Vibration and Associated Environments. Department of Defense, August 1960.

ABSTRACT

The David Taylor Model Basin has recently revamped its methods of handling large quantities of shock data. New methods incorporate high-speed digital computer techniques and types of components which may be useful to others involved in correcting, reducing, and interpreting large quantities of transient data. Computer programs for computing the response of mechanical systems to transient inputs, e.g., shock spectra, are also available.

Beal, T.R., "Finite Difference Analysis of the Shock Response of a Packaged Missile with Non-Linear Suspension Mounts". Lockheed Missiles and Space Division Report LMSD-703088, ASTIA AD243856, September 1960.

ABSTRACT

Physical phenomena are in most cases described by nonlinear differential equations. Because of the inherent difficulty in solving such equations, the general practice is to approximate the nonlinear functions by linear functions, thereby forming easily solvable equations which in many cases give sufficiently accurate results. It is the purpose of this study to present a method for handling nonlinear differential equations by the use of finite difference equations. The basic principles involved are well-known and may be found in any standard text on numerical analysis.

Brooks, R.O., "The Use of Graphic Techniques to Analyze Shock Motions of Lightly Damped Linear Spring Mass Systems", Bulletin 33, Shock, Vibration and Associated Environments. Department of Defense, Feb. 1964.

ABSTRACT

The "Phase-Plane Method" is used to obtain the response of a general two-degree-of-freedom system. Examples are presented utilizing linear spring-mass systems with either coulomb or viscous damping.

Brown, D., "Digital Techniques for Shock and Vibration Analysis" Presented at the SAE Meeting in Los Angeles, October 1962.

ABSTRACT

Numerical filtering is the smoothing of data by convolution of sampled data with a set of linear weights. Some novel applications of this technique to shock and vibration data analyses and impedance measurement are discussed and examples from existing programs are presented.

Cavanaugh, Richard, "Shock Spectra." 1964 Proc. IES, April 1964.

ABSTRACT

This paper illustrates how minor changes in the time-history of a pulse can cause major changes in the shock spectra. The seemingly minor transient at the tail end of the pulse causes gross changes in both the primary and residual spectra.

Crede, Charles E., "Shock Isolation of Structure Contents", Bulletin 32, Shock, Vibration and Associated Environments. Department of Defense, December 1963.

ABSTRACT

Shock isolation of equipment attached directly to underground structures can be accomplished by introducing a relatively compliant element between equipment and structure. This element is discussed by developing the theory of damage to be sustained by the equipment, theory of shock isolation, and the design of the isolator.

Cunniff, P.F., "A Graphical-Numerical Method for the Transient Response of Nonlinear Systems", NRL Report 5785, June 1962.

ABSTRACT

A graphical-numerical method is developed for determining the response of nonlinear systems subjected to transient loads. The method consists of constructing phase-plane trajectories in conjunction with simple computations. The numerical equivalent procedure of the graphical-numerical method is derived and a scaling law is presented for nonlinear multi-degree-of-freedom systems. Example problems are solved by the graphical-numerical method and the solutions compared with those of an analog computer at NRL. The solution of an example problem by both the graphical-numerical method and the numerical equivalent method indicates that the drawing error due to the graphical construction technique is negligible.

Fowler, Wallace, "An Analytical Study of an Undamped Nonlinear Single-Degree-of-Freedom System Subjected to Impulse Loading". University of Texas Structural Mechanics Research Laboratory, Army Contract No. DA19-129-QM-1383, August 1961.

ABSTRACT

The response to impulsive loading of an umdamped single-degree-of-freedom system with a yielding spring is studied. Primary interest is centered on the relationships between the permanent deformation produced by various types of impulsive loading and the parameters which characterize the acceleration pulse.

Fung, Y.C. and Barton, M.V., "Some Shock Spectra Characteristics and Uses". Presented at the ASME Meeting in Los Angeles, September 1958.

ABSTRACT

In this paper, applications of the shock spectrum are discussed. In particular, it is shown that, if the fundamental frequency (f_1, cps) of the structure is sufficiently high, a close approximation of the peak response of a multi-degree-of-freedom system can be obtained by the algebraic sum (not the sum of absolute values) of the peak responses of the individual degrees of freedom. Numerical results for a uniform cantilever beam subjected to a shock load uniformly distributed over its span show that the high-frequency requirement is satisfied if $2f_1t_m \gtrsim 1$, where t_m(sec) is the rise time of the pulse.

Fung, Y.C. and Barton, M.V., "The Performance of Nonlinear Systems Subjected to Ground Shock". Aerospace Eng., Vol. 21, No. 2, February 1962.

ABSTRACT

This paper offers a systematic treatment of the response of a nonlinear spring-mass system to ground shock. In particular, the question is asked in what manner, from the point of view of shock isolation, a nonlinear system offers any advantage over an optimally designed linear system.

The central point of the analysis is the application of the concept of "loading ratio" introduced in an earlier paper. The system considered here consists of a mass supported by a nonlinear spring, which may be either elastic or inelastic. The motion of the point of support induces motion of the sprung mass. The displacement and acceleration of the mass, and the strength and stiffness of the spring, are of particular interest.

Fung, Y.C. and Barton, M.V., "Shock Response of a Nonlinear System". J. Appl. Mech. Vol. 29, Series E, September 3, 1962.

ABSTRACT

The dynamic response of a nonlinear, single-degree-of-freedom spring-mass system to shock loading is investigated. A specific value of the maximum deflection is chosen as a characteristic quantity called the "maximum allowable deflection" — which forms the starting point for a simplified presentation of the analytical results. The shock-response characteristics of each nonlinear system is compared with that of an associated linear system. In particular, the ratio of the magnitudes of the loads applied as a pulse to produce the same maximum allowable deflection in a nonlinear system and its associated linear system is defined as the "loading ratio". This loading ratio, in combination with the usual "shock spectrum" for a linear system, can be used to determine the maximum dynamic load which a nonlinear system can withstand without exceeding the allowable deflection. A number of other engineering applications will be discussed also.

Fung, Y.C., Barton, M.V. and Young, D., "Response of Nonlinear System to Shock Excitation". Presented at the SAE Meeting in Los Angeles, October 1962.

ABSTRACT

Protection of structures, machinery, or equipment against ground shock is usually achieved by inserting a spring between the mass and the "ground", and often nonlinear springs are used. To evaluate performance of this mechanical system, maximum responses are presented in the form of shock response spectra. Extension of the shock spectra concept to nonlinear systems is straightforward; but owing to the inapplicability of the principle of superposition, special devices are needed to obtain nondimensional response spectra that are independent of the amplitude of the response. These non-dimensional spectra and their applications to analysis and design problems are discussed here.

Lane, D.W., "Digital Shock Spectrum Analysis by Recursive Filtering", Bulletin 33, Shock, Vibration and Associated Environments. Department of Defense, February 1964.

ABSTRACT

An effecient method of digital shock spectrum analysis employs recursive filtering, a feedback technique which is readily developed using Z transform techniques. By this means, the accuracy, flexibility and production capability of the digital computer are made available without the sacrifice of excessive computer time.

Lee, S.Y., "Indeterminate Shock Analysis", Bulletin 30, Shock, Vibration and Associated Environments. Department of Defense, May 1962.

ABSTRACT

This paper presents a practical method of designing and testing for shock and indeterminate shock loadings. It is based on part of the results of the writer's research project "New Approaches in Shock Analysis" at the University of Southern California in 1959. The rigorous mathematics and theories involved have been greatly simplified to aid in understanding the method.

Lloyd, W., "Practical Aspects of Digital Spectral Analysis". Royal Air-Craft Establishment Technical Note No. Space 11, ASTIA AD284243, May 1962.

ABSTRACT

Two of the commonest methods of computing spectra are explained briefly, together with the relation between resolution, length of sample and error in the estimate.

Emphasis is laid on the desirability of planning an experiment through to the end of the data processing before starting to record information. Types of equipment available for recording and digitising are tabulated and ways of choosing length of sample and sampling rate are indicated, together with some advantages to be gained by the use of digital filters.

Luhrs, H.N. and Spence, H.R., "Influence of Shock Machine Loading on Shock Spectra". Report No. TN61-31, Space Technology Laboratory, ASTIA AD260308, May 1961.

ABSTRACT

The influence of shock machine loading, by the test item, was investigated to determine whether any peaks or notches are introduced into the acceleration shock spectrum which would cause excessive over or under testing at various frequencies. The results showed that the shock spectrum for a terminal peak saw-tooth pulse is quite insensitive to shock machine loading. The very low frequency end (below 100 cps for a 6 millisecond pulse) is most influenced, whereas the higher frequencies are virtually not influenced at all. The results also showed that even for a very high loading condition the primary spectrum is in all cases less severe than the residual spectrum.

Luke, Robert R., "The Impact Response of Single-Degree-of-Freedom Systems With Viscous Damping", University of Texas Structural Mechanics Research Laboratory, Army Contract No. DA19-129-QM-1383, ASTIA AD246942, June 1960.

ABSTRACT

As a first step in the analysis and understanding of the response of more complex systems subjected to impact, the impact response of a single-degree-of-freedom system with viscous damping is determined and presented in the form of displacement and acceleration shock spectra. It is assumed that displacement and acceleration magnitudes are criteria for damage due to impact and that the dynamic load factor is a measure of the severity of the shock to which the system is subjected.

Shock spectra are presented for six differently shaped acceleration pulses to show, by comparison, the effect of pulse shape on the maximum response of the system. In addition, the effects of damping, the ratio of the pulse duration to system natural period, pulse amplitude, and total impulse on the maximum system response are presented.

Marous, J.J. and Schell, E. H., "An Analog Computer Technique for Obtaining Shock Spectra", Bulletin 33, Shock, Vibration and Associated Environments. Department of Defense, February 1964.

ABSTRACT

A unique method of computing the shock spectra of transient motions on an analog computer has been developed by the Flight Dynamics Laboratory in cooperation with the Directorate of Systems Dynamic Analysis of the Air Force Research and Technology Division. The uniqueness of the method is largely due to controlling the output voltages in such a manner that a direct graphical plot of the shock spectrum is recorded on a strip chart recorder.

Mains, R.M., "Structural Response to Dynamic Load", Bulletin 30, Shock, Vibration and Associated Environments, Part II. Department of Defense, January 1962.

ABSTRACT

This paper represents an attempt to generalize the problem of calculating responses to random vibration and shock into a set of simple principles, which are sufficient to produce numerical solutions to practical problems. Within the limitations of superposition and linearity, there are no restrictions on the recipes given so that the methods are general.

McCool, W.A., "A Digital Computer Program for The Analysis of Recorded Shock Motions". White Sands Missile Range, Flight Simulation Laboratory Data Report NR23, New Mexico, May 1961.

ABSTRACT

The digital computer program described in this paper mechanizes a procedure which facilitates the systematic analysis of recorded shock motions. With a given shock motion as an input, which can be either a velocity or an acceleration record, the computer program generates a "shock spectrum" which provides a means for evaluating the damage potential of the shock to physical structures.

Morrow, Charles T., "The Shock Spectrum", Electrical Manufacturing, August 1959.

ABSTRACT

A basic discussion on the use of shock spectra as a design parameter. How the shock spectrum is a means of stating mechanical shock requirements and why the use of the shock spectrum is preferred to the pulse shape for specifying shock test requirements.

Nash, William A. and Hijab, Wasfi A., "On Generalized Shock Spectra and Response Surfaces", Bulletin 23, Supplement to Shock, Vibration and Associated Environments. Department of Defense, June 1956.

ABSTRACT

The usual concept of the shock spectrum as an index of the damaging capacity of a shock is extended to take account of the effects of response accelerations other than the maximum usually employed. Also, recently developed statistical methods are used to aid in the design of experiments intended to indicate the response surface corresponding to a given shock.

O'Hara, George, "Effect Upon Shock Spectra of the Dynamic Reaction of Structures", NRL Report 5236, December 1958.

ABSTRACT

Attempts to use fiducial limit curves of a set of classes of shock spectra as a basis for the design of structures have shown that the design spectra obtained by the combinatorial analysis of many shock spectra tend to be overconservative. This interim report presents a possible explanation for this. It exhibits some experimental evidence to show that the values of interest in a shock spectrum plot tend to lie in the valleys of that plot and not upon the peaks, whereas fiducial limit curves are controlled by the peaks of the individual shock spectra.

O'Hara, G.J., "A Numerical Procedure for Shock and Fourier Analysis", NRL Report 5772, June 1962.

ABSTRACT

This paper describes and derives a numerical integration technique presently being used at NRL for problems in structural dynamics. The method is capable of computing damped and undamped shock spectra, Fourier sine, cosine, and regular Fourier integrals (frequency response from transient response), and the inversion of the Fourier sine and cosine integrals (transient response from frequency response). It can also be used to calculate Fourier series coefficients, and for the numerical solution of nonlinear equations. Several examples are worked out in detail and some others calculated by NAREC (NRL's digital computer) are shown to present some idea of the precision of the method. This report does not assume a sophisticated mathematical background and uses only those techniques which are available to undergraduate students in engineering.

O'Hara, G.J. and Cunniff, P.F. "A Numerical Method for the Transient Response of Nonlinear Systems", NRL Report 5917, June 1963.

ABSTRACT

Numerical integration equations are derived for determining the response of nonlinear systems subjected to transient loads. The numerical method consists of approximating the nonlinear variables and the forcing functions in the differential equations over a short interval of time by their mean value, by a straight line, or by a parabola. This allows for Duhamel integral type solutions for the nonlinear terms. A step by step solution follows which uses an iteration method during each increment of the solution. The sufficient condition for the convergence of the iteration method is presented for the case of N numerical equations. A scaling law is presented which eliminates linear damping from the equation of motion by a prescribed transformation. Example problems of a one-degree-of-freedom system and a two-degree-of-freedom system are solved by the numerical integration equations and the solutions are compared with response curves obtained from analog computers at NRL.

Pittman, R. and Wheeler, R.W., "Solution of Structural Response Problems by Analog Computers". Bulletin 33, Shock, Vibration and Associated Environments. Department of Defense, February 1964.

ABSTRACT

Examples are presented to illustrate that analog computers are ideally suited for the solution of structural response problems. Parameter changes are easily made and the solution is readily obtained in a plotted form. Nonlinear and highly redundant structures and the various forms that forcing functions take do not represent obstacles in the determination of a structure's dynamic response characteristics.

Ripperger, E.A. and Fowler, W.T., "The Response of Yielding Structures to Shock Loading", Bulletin 30, Shock, Vibration and Associated Environment. Department of Defense, February 1962.

ABSTRACT

A theoretical and experimental study of the response of a tip-loaded, mild, steel cantilever beam to acceleration pulses applied at the root is described. The relationship between permanent deformation of the beam, and the characteristic features of the acceleration pulse is described. It is found that within certain prescribed limits, the peak acceleration, the pulse duration, and the natural period of the beam are all equally effective in producing permanent deformation. Pulse shape is also shown to be a significant factor.

Schmidt III, W.E., "Use of the Analog Computer to Study Cushion Characteristics and Package Design", Bulletin 31, Shock, Vibration and Associated Environments. Department of Defense, March 1963.

ABSTRACT

This paper describes the force analyzer and the analog computer which have been designed specifically for package and cushion analysis.

Barnes, Ward P., "A Spectral Analyzer for Shock Environment", Bulletin 29, Shock, Vibration and Associated Environments. Department of Defense, June 1961.

ABSTRACT

Recent military specifications require that a shock environment be described in terms of a shock-response spectrum rather than the traditional terms of intensity, duration and waveform. This paper describes a new method of determining the response to a shock pulse, the analysis equipment, and the results of analyses of both test shocks and actual shock environments.

Thomson, W.T., "Shock Spectra of a Nonlinear System", J. Appl. Mech. Vol. 27, Series E., No. 3, September 1960.

ABSTRACT

The peak response under pulse excitation, of a single-degree-of-freedom system with nonlinear spring approximated by two straight lines, is analytically determined. Shock spectra for the step and delta function are developed with damping as parameter. For the rectangular pulse excitation, the shock spectra for zero damping are presented with pulse duration as a parameter. For the arbitrary pulse shape, a general procedure is discussed and illustrated for the sine pulse of differing pulse duration. Generalized phase-plane plots associated with the problem are included. The shock spectra presented are applicable only when the first peak corresponds to the maximum response thereby avoiding the questions of unloading.

Todaro, A.F., "Shock Spectra for a General Forcing Function", Bulletin 33, Shock, Vibration and Associated Environments. Department of Defense, February 1964.

ABSTRACT

Shock spectra, which are the maximum responses of many one-degree-of-freedom systems to a particular shock force or motion, can be helpful in the understanding of the general motion of elastic bodies subjected to shock forces or motions. While shock spectra for well-defined analytic functions are available, those for non-analytic forcing functions are not so easily found. A method for determining these spectra is described.

Wah, T., Rastrelli, L.U., Basdekas, N.L. and DeHart, R.C., "Response of Missile Structure to Impulse Loading", Wright-Patterson AFB Aeronautical Systems Division Technical Document Report ASD-TDR-62-475, ASTIA AD403702, March 1963.

ABSTRACT

Procedures for analytically predicting the response of missile bodies to blast loadings are presented. The investigation involves the behavior of cylindrical shells (with various end-closures) and circular, flat plates. The numerical results obtained from the analytical methods compare favorably with the experimental data required during the study.

B-4 - Impedance

Belsheim, R.O., "Introductory Remarks", Bulletin 30, Shock, Vibration and Associated Environments, Department of Defense, January 1962.

ABSTRACT
These papers have been arranged to present, in a "tutorial" sense, the concept, determination, and application of what may be referred to as mechanical impedance procedures. Some historical and introductory material is presented to prepare the way for the papers.

Belsheim, R.O. and Young, Jr., J.W., "Mechanical Impedance as a Tool for Shock or Vibration Analysis", NRL Report 5409, February 1960.

ABSTRACT
This report introduces mechanical impedance to those already familiar with classical vibration theory. The impedance term is defined and discussed in detail. The analog between mechanical and electrical systems is noted and electrical circuit theorems which are especially applicable to mechanical systems are presented. In order to provide an introduction to the impedance concept, several simple mass-spring-dashpot systems are analyzed to obtain their impedance and the results are presented graphically. Since most structures are too complicated to yield to an analytical determination of impedance, methods for experimentally measuring mechanical impedance are discussed. Applications of impedance techniques for the general analysis of some shock and vibration problems are discussed.

Blake, Ralph E., "Applications of Impedance Information", Bulletin 30, Shock, Vibration and Associated Environments, Department of Defense, January 1962.

ABSTRACT
Only a few applications of impedance information have yet been made to the solution of engineering problems. However, engineering research is being carried on by several groups to develop methods, apparatus, and theorems for application to some important problems in shock and vibration. The greatest current effort is on developing more effective sound and vibration isolation systems. Improvements in methods of making field measurements of dynamic environment and methods of simulation in the testing laboratory are also being studied. Ultimately, any area of shock and vibration work which deals in complicated linear systems should benefit from applications of the techniques and knowledge being developed.

McCalley, Jr., Robert B., "Velocity Shock Transmission in Two Degree Series Mechanical Systems", Bulletin 23, Supplement to Shock, Vibration and Associated Environment. Department of Defense, June 1965.

ABSTRACT

Previous work by Mindlin and Crede on shock transmission through isolators assumed that the mass of the element was negligible in comparison to the mass of the chassis upon which it was mounted. A more exact analysis of the problem is given without any restriction on the relative masses of the element and the chassis. Contour map charts for (a) shock transmissibility through the isolator and (b) relative magnification of the chassis deflection have been prepared with a range of 10^6 in both spring and mass ratios for an undamped system.

Blake, R.E. and Ringstrom, T., "The Influence of Mass and Damping on the Response of Equipment to Shock and Vibration", Bulletin 28, Shock, Vibration and Associated Environments. Department of Defense, August 1960.

ABSTRACT

Much present practice in designing for shock and vibration environments is highly conservative because impedance effects have largely been neglected. Theoretical results are reported on the amount of reduction to be expected from such effects and the way in which mass, natural frequency, and damping will influence design stress.

Mustain, Roy W., "Application of Mechanical Admittance Data to the Solution of a Practical Problem", Bulletin 30, Shock, Vibration and Associated Environments. Department of Defense, January 1962.

ABSTRACT

This paper describes the practical application of mechanical admittance (reciprocal of impedance) data to the numerical solution of a vibration isolation problem. The solution results in the selection of the most favorable vibration isolator to protect rigid equipment from dynamic excitations of a nonrigid supporting structure. The solution is presented in steps and produces optima values of stiffness and damping for the selected vibration isolator. An envelope of transmissibility curves for all permitted system designs is established.

O'Hara, G.J., "Impedance and Shock Spectra", J. Acoust. Soc. Amer., Vol. 31, No. 10, October 1959.

ABSTRACT

This paper examines the theory of shock spectra, Fourier spectra, and mechanical impedance to determine qualitatively the effect upon shock spectra of the dynamic reaction of structures. The concept of shock spectrum dip is presented, explained, and experimental verification exhibited. The potentially extreme overconservatism in design resulting from incorrect usage of shock spectra is pointed out.

On, F.J., "A Theoretical Basis for Mechanical Impedance Simulation in Shock and Vibration Testing", Bulletin 33, Shock, Vibration and Associated Environments. Department of Defense, March 1964.

ABSTRACT

Fundamental and useful mathematical relations for impedance are given for the analysis, control, and simulation in mechanical structures that can be considered as one-dimensional linear-passive systems. The validation of these expressions in the application to impedance control and simulation is approached primarily from analytical considerations, with general theories of application summarized.

Plunkett, Robert, "Analytical Determination of Mechanical Impedance", Bulletin 30, Shock, Vibration and Associated Environments, Department of Defense, January 1962.

ABSTRACT

This paper discusses the impedance, dynamic stiffness, effective mass, mobility and receptance of simple lumped elements and shows how they may be combined to find the impedance of systems of moderate complexity. It also indicates methods for finding the impedance of simple, uniform, continuous systems and lists references dealing with more complex structures.